OH: Fridays 10:30 - 12:30

today: radiative transfer eq^n, want specific intensity

in a vaccuum, intensity is constant along ray

$$\frac{dI_x}{ds} = 0$$

S: geometrical path length

constancy implies $\longrightarrow$ $F \sim \frac{1}{r^2}$ (flux)

why is night sky dark?
$\longrightarrow$ Homer's Paradox

In matter, energy can be ① removed, ② added, & ③ redirected

blue photons from Sun get scattered $\rightarrow$ blue sky!

w/o scattering ③, can modify rad. transfer eq^n

absorption coefficient: $\alpha = n\,\sigma_\nu$ (# density, cross section)

$\alpha = \rho\,\kappa_\nu$ (density, opacity)

$[\alpha] = L^{-1}$

prob. of getting absorbed per unit length

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

emmission coefficient

$[j_\nu] = E\,L^{-1}$

energy lost per unit length

prob. of atom changing energy levels

$\alpha \in$ true absorption & stimulated emmission

$\Rightarrow$ proportional to # of photons

Elementary sol^n's

① emmission only: $\int \frac{dI_\nu}{ds} = \int j_\nu$

$$I_\nu(s) = I_{\nu,0}(s) + \int_{s_0}^{s} j_\nu(s')\,ds'$$

sum of all things added

② absorption only:

$$\frac{dI_\nu}{ds} = -\alpha I_\nu$$

$$I_\nu(s) = I_{\nu,0}(s) \cdot \exp\left(-\int_{s_0}^{s} \alpha_\nu(s')\,ds'\right)$$

new variable!

$$d\tau = \alpha ds \rightarrow \tau = \int_{s_0}^{s} \alpha(s')ds'$$

tau

optical depth

$[\tau] = 0$

$$\frac{d}{d\tau}[I_\nu] = -I_\nu + S_\nu$$

$\tau > 0 \rightarrow$ optically thick, absorped

$\tau < 0 \rightarrow$ optically thin, adding energy

$S_\nu = \frac{j_\nu}{\alpha_\nu}$ — emission: W cm^{-1} Hz^{-1}; absorption: cm Hz^{-1}

source fn

property of conditions of medium

$$I_\nu(\tau) = I_\nu(0)e^{-\tau} + \int_0^\tau e^{-(\tau-\tau')} S(\tau')d\tau'$$

example: S is constant $\rightarrow$ $\tau > 0$

$$I_\nu(\tau) = S_\nu + e^{-\tau}(I_\nu(0) - S)$$

if $I_\nu(0) > S \rightarrow \frac{dI}{dS} < 0$ — I decreases along a ray

if $I_\nu(0) < S \rightarrow \frac{dI}{dS} > 0$ — I increases along a ray

conclusion: $I_\nu \rightarrow S$ as $\tau \rightarrow \infty$

temp characterizes thermal equillibrium of objects

Thermal Radiation & Blackbody Radiation

Radiation emmitted by matter in thermal equilibrium

$S = \frac{j}{\alpha}$ property of medium & material

Source function is universal f^n of temp only: $S = B_\nu(T)$
same functional form

Radiation that is in thermal equilibrium w/ matter

only leave material if in equilibrium

enclosed within matter, bounces around until it is in thermal equilibrium
intensity is f^n of temp only: $I_\nu = B_\nu(T)$
$W cm^{-2} Hz^{-1} sr^{-1}$

thermal rad. tends to BB rad. w/ $\tau \to \infty$

Being in equillibrium: everything has same avg KE across all matter

Temp is f^n of KE

distribution for these particles → must assume Maxwell-Boltzmann statistics
→ velocity! (Maxwellian) E decreases exponentially

Cool, we know for the matter. What about for the photons?

What's distribution for the photons?

Rayleigh: equipartition of radiation & atoms

Planck: h energy to frequency

Einstein: assume gas has ground & 1st excited state
atoms can jump between

excited → ground emmission

assign % that if photon comes, may get absorbed

didn't get Plancks f^n!

assign % of ground → excited proportional to intensity
stimulated emmission

emotional hair

20 yrs later

discrete energy levels

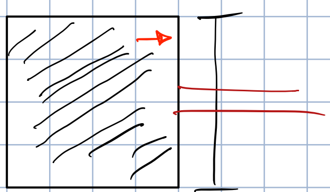
Indian prof: assumed photons are indistinguishable → can't tell apart — did something to Planck's f^n? ↔?

Statistical mechanics

Bose: publish Einstein's work in English

Bose-Einstein vs Fermi-Dirac statistics

Derivation using thermal equilibrium



heat, energy, pressure, volume

$$dQ = dU + pV \quad \to \text{1st law thermo.}$$

$$dS = \frac{dQ}{T} \quad \to \text{2nd law thermo.}$$

entropy

$u = u(T)$ — energy/unit volume

radiation: $U = uV$ & $p = \frac{u}{3}$ — energy density

$$\boxed{du = \frac{du}{dT}\,dT \quad \text{b/c } u(T) = u}$$

$$dS = \frac{1}{T}\,d(uV) + \frac{u}{3T}\,dV = \frac{1}{T}\left(u\,dV + V\frac{du}{dT}\,dT\right) + \frac{u}{3T}\,dV$$

$$= \frac{V}{T}\frac{du}{dT}\,dT + \frac{4u}{3T}\,dV$$

b/c dS is perfect differential

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{V}{T}\frac{du}{dT} \quad \& \quad \left(\frac{\partial S}{\partial V}\right)_T = \frac{4u}{3T}$$

if scalar Q exist s.t. $\frac{\partial}{\partial x}(Q) = A$, $\frac{\partial}{\partial y}(Q) = B$

$\to dQ = dx\,A + dy\,B = \nabla Q\,dx$

assume $\frac{\partial^2}{\partial x \partial y} Q = \frac{\partial^2}{\partial y \partial x} Q$

$$\frac{\partial^2}{\partial T \partial V} \quad \text{work} \quad \vdots \quad \frac{\partial^2}{\partial V \partial T}$$

$$\frac{du}{dT} = \frac{4u}{T}$$

$$\frac{du}{u} = 4\frac{dT}{T}$$

$\ln(u) = 4\ln(T)$

$\ln(u) = \ln(T^4)$

$u = T^4 \cdot a$ — a is proportion constant for radiations

$$\to u(T) = a\,T^4 \quad \text{Stefan–Boltzmann law}$$

$$\to F^{flux} = \sigma T^4$$