Due Sunday Homework

Thermal Radiation: $S = \frac{j_\nu}{\alpha_\nu} = B_\nu(T)$

$I_\nu \rightarrow S_\nu$ for $\tau \rightarrow \infty$

Blackbody Radiation: $I_\nu = B_\nu(T)$

$u$ - internal energy

$u = aT^4$

$F = \sigma T^4$ Stephan Boltzmann law

change in entropy as $f^n$ of volume

$$\int \left(\frac{\partial S}{\partial V}\right)_T = \frac{4u}{3T} = \int \frac{4aT^3}{3}$$

@ $T=0$, $S=0$

$$S = \frac{4aT^3V}{3} + \cancel{C}$$

For adiabatic process (preserve entropy, can't be discontinuous)

$S$ = constant entropy of black body radiation

$\rightarrow T^3V = \frac{3}{4a}$ = constant

$\rightarrow PV^{4/3}$ = constant $\quad 4/3 \rightarrow \gamma$

$\gamma_a = 4/3$ for radiation perfect gas: $\gamma_a = 5/3$

sets max luminosity for stars

very luminous? $\rightarrow$ hella radiation, not lot gas

can't have stars too bright

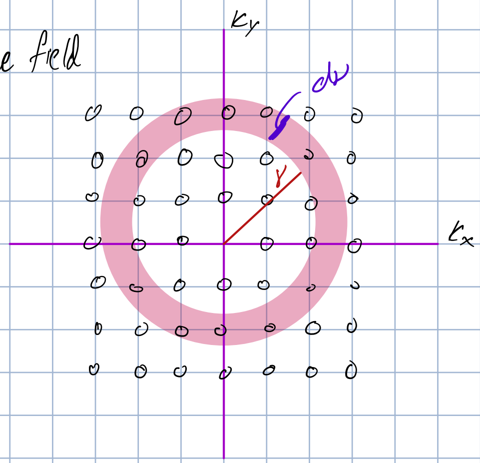
Fluidizing eq^n's

Avogadro's # very large
density can be seen as a fluid/continuum

Planck: quantized/packaged radiation
thought it was just a math trick

Want intensity as f^n of frequency

lattice field



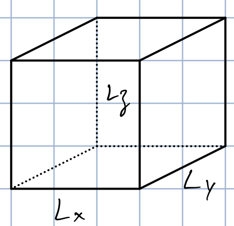

$k^2 = k_x^2 + k_y^2$

each is a sol^n to wave eq^n

density of states

$\rho = 2\pi\nu \, d\nu$

avg energy in this small volume
old: equipartition energy
if keep going, that's a rip




standing waves

$E = h\nu$

$k = \frac{2\pi\nu}{c} \cdot n$

$n = (n_x, n_y, n_z)$

2 polarizations

$n_x = \frac{L_x}{2\pi} k_x$

discrete change

$\Delta n_x = \frac{L_x}{2\pi} \Delta k_x$

in 3D! $\Delta N = \Delta n_x \Delta n_y \Delta n_z$

$\Delta k_x \Delta k_y \Delta k_z = d^3k$

$\Delta N = \frac{L_x L_y L_z}{(2\pi)^3} d^3k$

what is $d^3k$? $\rightarrow$ $d^3k = k^2 dk \, d\Omega$

$= \frac{(2\pi)^3}{c^3} \cdot \nu^2 \, d\Omega$

???

$V = L_x L_y L_z$

$\Delta N = \frac{V}{(2\pi)^3} \cdot \frac{(2\pi)^3}{c^3} \nu^2 d\Omega$

$$= \frac{V}{c^3} \nu^2 d\Omega$$

2 polarizations

density of states: $\quad f_s = \frac{2\nu^2}{c^3} \qquad \frac{\Delta N}{V}$

$c = \lambda \nu$

$E_n = nh\nu \quad \to E_1, E_2, \dots$ discretize photons

$\beta = \frac{1}{kT}$

$$P(E_n) = \frac{e^{-\beta E_n}}{\sum_{n=0}^{\infty} e^{-\beta E_n}}$$

→ chance of $E_n$

→ total

normalized!

↳ probability of seeing $E_n$

→ overbar means average

$$\frac{f'(x)}{f(x)} = \frac{d}{dx}(\ln(f))$$

$$\overline{E_\nu} = \sum_{n=0}^{\infty} E_n P(E_n)$$

$$= \frac{\sum_{n=0}^{\infty} E_n e^{-\beta E_n}}{\sum_{n=0}^{\infty} e^{-\beta E_n}}$$

$$\sum_{n=0}^{\infty} e^{n \cdot \frac{h\nu}{kT}} = \frac{1}{1-\exp(\frac{h\nu}{kT})}$$

$$= -\frac{\partial}{\partial\beta}\left( \ln\left( \sum_{n=0}^{\infty} e^{-\beta E_n} \right)\right)$$

geometric series w/ ratio $\frac{h\nu}{kT}$

$$-\frac{\partial}{\partial\beta}\left( \ln\left( \frac{1}{1-e^{-\beta E_n}} \right)\right)$$

$$-\frac{\partial}{\partial\beta}\left( \ln(1) - \ln(1-e^{-\beta E_n}) \right)$$

$$\frac{\partial}{\partial\beta}\left( \ln(1-e^{-\beta E_n}) \right)$$

$$\frac{1}{1-e^{-\beta E_n}} \cdot \frac{\partial}{\partial\beta}(1-e^{-\beta E_n})$$

$$\frac{1}{1-e^{-\beta E_n}} \cdot E_n e^{-\beta E_n}$$ bring to denominator

$$\frac{E_n}{e^{\beta E_n}(1-e^{-\beta E_n})}$$

$$\frac{E_n}{e^{\beta E_n}-1}$$

$$\overline{E_\nu} = \frac{h\nu}{\exp(\frac{h\nu}{kT})-1}$$

$u_\nu(\Omega) = f_s \cdot \overline{E_\nu}$    avg density is density of one state times avg energy

$$u_\nu(\Omega) = \frac{2\nu^2}{c^3} \cdot \frac{h\nu}{\exp(\frac{h\nu}{kT})-1}$$

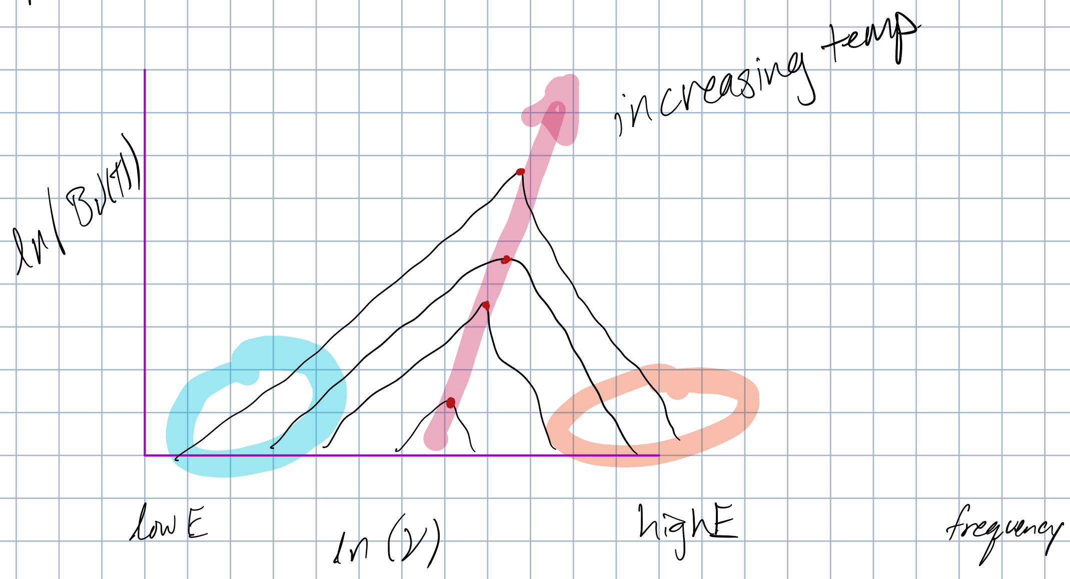
$$u_\nu(\Omega) = \frac{I_\nu(\Omega)}{c} \quad \to \quad I_\nu(T) = B(T)$$

$$B_\nu(T) = c \cdot u_\nu(\Omega)$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left( \frac{1}{\exp(\frac{h\nu}{kT})-1} \right)$$

B → spectral radiance

# Properties of Black Body Radiation




What we know before Planck

- at low Energy → Rayleigh Jeans law

in limit $kT >> h\nu$

$\hookrightarrow \exp\left(\frac{h\nu}{kT}\right) - 1 \approx \frac{h\nu}{kT}$

$$I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT$$

pretty good for most Radio Astronomy

- at high Energy → Wien's law

in limit $kT << h\nu$

$$I_\nu^{W}(T) = \frac{2h\nu^3}{c^2} \cdot \exp\left(-\frac{h\nu}{kT}\right)$$

- Monotonicity of T

$$B_\nu(T_1) > B_\nu(T_2) \quad \text{iff} \quad T_1 > T_2$$

$$\frac{\partial B_\nu(T)}{\partial T} > 0$$

· peak of distribution moves to right

let $\nu_{max}$ place where $\frac{\partial B(T)}{\partial \nu} = 0$

let $x = \frac{h\nu}{kT}$ → solve $x = 3(1-e^{-x})$

$$\nu_{max} = 2.82 \cdot \frac{kT}{h}$$

cf $\lambda_{max}$ $\nu_{max}$

$$u(T) = a\,T^4$$

$$F(T) = \sigma T^4$$

$$a = \frac{8\pi^5 k^4}{15 c^3 h^3}$$

$$a = \frac{4\sigma}{c}$$

fine transformation

$$x \to \alpha x$$

change space

$B(T)$ invariant but need to scale temperature

gravitational entropy

$$\frac{\partial B}{\partial \nu} = \frac{\partial}{\partial \nu}\left(\frac{2h\nu^3}{c^3} \cdot \frac{1}{\exp(\frac{h\nu}{kT})-1}\right)$$

$$= \frac{2h}{c^3} \cdot \frac{\partial}{\partial \nu}\left(\frac{\nu^3}{\exp(h\nu/kT)-1}\right)$$

$$= \frac{2h}{c^3} \cdot \frac{(e^{h\nu/kT}-1)(3\nu^2) - (\nu^3)(\frac{h}{kT}e^{\frac{h\nu}{kT}})}{(e^{h\nu/kT}-1)^2}$$

$$= \frac{2h}{c^3} \quad \frac{\nu^2\left[(e^{h\nu/kT}-1)3 - \nu\frac{h}{kT}e^{\frac{h\nu}{kT}}\right]}{(e^{h\nu/kT}-1)^2}$$

$$= \cancel{\frac{2h}{c^3}} \quad \frac{\cancel{\nu^2} \left( (e^x-1)3 - xe^x \right)}{\cancel{(e^x-1)^2}}$$

$$0 = 3(e^x-1) - xe^x$$

$$xe^x = 3(e^x-1)$$

$$x = 3(e^x-1)e^{-x}$$

$$x = 3(1-e^{-x})$$

$\longrightarrow$ $x = 2.81$ or $x = 0$

$$x = 2.81 = \frac{h\nu_{max}}{kT} \longrightarrow h\nu_{max} = 2.81\,kT$$