$I_\nu = \int ? \, d\Omega$

$J = \frac{1}{} \int \alpha$

$E = \frac{1}{c} \int$

3 processes

1.78 pg 32 :

$$\alpha_\nu = \frac{h\nu}{4\pi} n_1 B_{12} \left(1 - \frac{g_1 n_2}{g_2 n_1}\right) \phi(\nu)$$

$$S_\nu = \frac{2h\nu^3}{c^2} \left(\frac{g_2 n_1}{g_1 n_2} - 1 \right)^{-1}$$

No matter what

① Boltzman → Planck

② non Maxwellian just need n_1 & n_2 energy ratio

③ $n_2 > n_1$

Scattering

simplest isotropic coherent
frequency in = frequency out

scattering prob. is indpt. of direction

isotropic scattering ≠ isotropic radiation (intensity indpt. of direction)

$\frac{dI_\nu}{dS} = -\alpha_\nu I_\nu + j_\nu$

isotropic coherent scattering

equate absorbed to power avg intensity E

E/L

$j_\nu = J_\nu \cdot \sigma_\nu$

emission

scattering coefficient L^{-1}

$j_\nu / \sigma_\nu \to \quad S = J_\nu = \frac{1}{4\pi} \int I_\nu \, d\Omega$

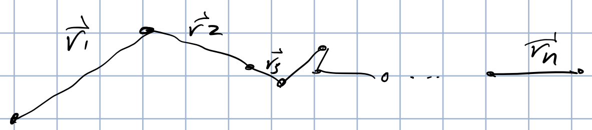
$\frac{dI_\nu}{dS} = -\sigma_\nu (I_\nu - J_\nu)$ integral differential eqn

path

optically thick → local
optically thin → everything is impt.

consider path as random walk

possible to estimate w/ random walks



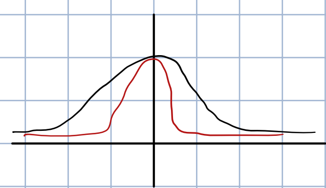
$$\vec{R} = \vec{r}_1 + \vec{r}_2 + \ldots + \vec{r}_N$$

$$\langle \vec{R} \rangle = 0 \quad \text{isotropic}$$

scalar avg.

$$\langle |\vec{R}|^2 \rangle = l_*^2$$
$$= \langle |\vec{r}_1|^2 \rangle + \langle |\vec{r}_2|^2 \rangle + \ldots + \langle |\vec{r}_N|^2 \rangle$$
$$+ 2\langle \vec{r}_1 \cdot \vec{r}_2 \rangle + 2\langle \vec{r}_1 \cdot \vec{r}_3 \rangle + \ldots$$
$$+ 2\langle \vec{r}_2 \cdot \vec{r}_3 \rangle + \ldots$$
$$+ \vdots$$

correlation of direction, but we don't care about direction → isotropic



Spread vs avg → variance
asymmetry w/ mean → skew
intermittency (offla) → curtosis

$$l_*^2 = N \cdot l^2$$

→ l^2 = typical displacement average mean free path

$$\boxed{l_* = \sqrt{N}\, l}$$

Optically thick w/ length L

\# scatterings required is $l_* \sim L \;\rightarrow\; N = \left(\frac{L}{l}\right)^2 \approx \tau^2$

↳ optical thickness of medium

Combined Scattering & Absorbtion

α_ν = absorption for thermal emission (something w/ temperature)
σ_ν = coherent isotropic scattering

$$\frac{dI_\nu}{ds} = -\alpha_\nu\underbrace{(I_\nu - B_\nu)}_{\text{along ray}} - \sigma_\nu(I_\nu - J_\nu)$$

emission, but can define source f^n
Source f^n is BB

$$\frac{dI_\nu}{ds} = -\underbrace{(\alpha_\nu + \sigma_\nu)}_{\to \text{ net absorption}}(I_\nu - S_\nu)$$

$$S_\nu = \frac{\alpha_\nu B_\nu + \sigma_\nu J_\nu}{\alpha_\nu + \sigma_\nu}$$

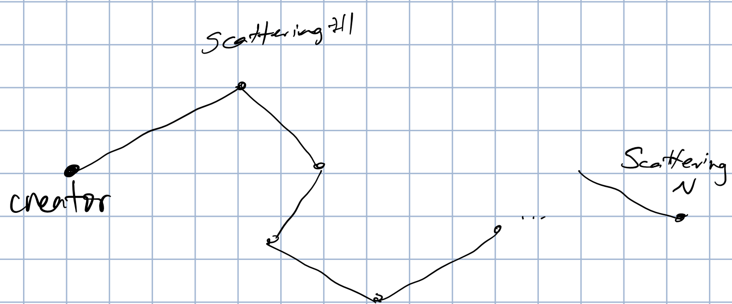
define optical depth $d\tau_\nu = (\alpha_\nu + \sigma_\nu)\, ds$

$$l_\nu = (\alpha_\nu + \sigma_\nu)^{-1}$$

probability freepath end w/ true absorption

$$\epsilon_\nu = \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu}$$

$$1 - \epsilon_\nu = \frac{\sigma_\nu}{\alpha_\nu + \sigma_\nu} \qquad \text{prob of scattering}$$



typical length of a path $N = \epsilon^{-1} \to l = \sqrt{N}\, l_*$

$$\to l_* = \frac{l}{\sqrt{\epsilon^{-1}}}$$

$$\to l_* = (\alpha_\nu(\alpha_\nu + \sigma_\nu))^{-1/2}$$

"diffusion length"
"thermalization length"

Medium of finite length L

$$\tau_* = \frac{L}{l_*}$$

optically thin $\tau_* \ll 1$
optically thick $\tau_* \gg 1$

Monochromatic luminosity

thin → nothing absorbed or scattered

$$L = 4\pi \alpha_\nu B_\nu V$$

J   volume

thick $\tau >> 1$

$$L = 4\pi \alpha_\nu B_\nu A \cdot l_*$$

area   effective free path