$\vec{F} = m\vec{a}$

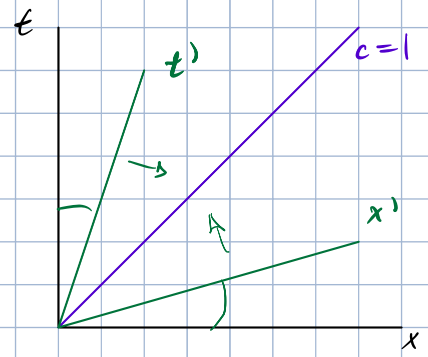
$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = 0$$

$$\vec{x}' = x - vt$$
$$t' = t$$

$$x' = \gamma(x - vt)$$ "x boost"
$$t' = \gamma\left(t - \frac{vx}{c}\right)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \beta = \frac{v}{c} \qquad |\beta| < 1$$

$$s^2 = (ct)^2 - |\vec{x}|^2$$



Spacetime

"Seeing" is special

Elementary consequences

① LF contraction

I am rest watching in stadium, polevaulter has own frame

$$L_0 = (x_2' - x_1') = \gamma(x_2 - x_1) = \gamma L$$

$$L = \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} L_0 \leq L_0$$

moving rods get shorter

② Time Dilation

$$T = (t_2 - t_1) = \gamma(t_2' - t_1') = \gamma T_0 > T$$

runner says 30 sec
I say 1 min

moving clocks run slow

③ Transformation of Velocities

define velocity in my frame

$$dx = \gamma\,(dx' + v\,dt')$$

$$dt = \gamma\left(dt' + \frac{v\,dx'}{c^2}\right)$$

divide top & bottom by dt'

$$u_x = \frac{dx}{dt} = \frac{\gamma\,(dx' + v\,dt')}{\gamma\left(dt' + \frac{v\,dx'}{c^2}\right)} \overset{*}{=} \frac{u_x' + v}{1 + \frac{v}{c^2}\cdot u_x'}$$

$$u_y = \frac{dy}{dt} = \frac{dy'}{\gamma\left(dt' + \frac{v\,dx'}{c^2}\right)} \overset{*}{=} \frac{u_y'}{\gamma\left(1 + \frac{v\,u_x'}{c^2}\right)}$$

split into $\parallel$ & $\perp$ components

$$\vec{u}_{\parallel} = \frac{u_{\parallel}' + v}{1 + \frac{v\,u_{\parallel}'}{c^2}} \qquad \vec{u}_{\perp} = \frac{\vec{u}_{\perp}'}{\gamma\left(1 + \frac{v\,u_{\parallel}'}{c^2}\right)}$$

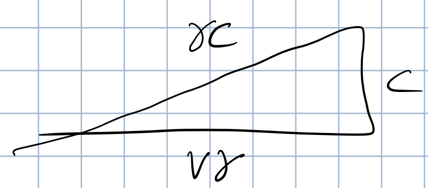
$$\tan\theta = \frac{u_\perp}{u_\parallel} = \frac{u'\sin\theta'}{\gamma\,(u'\cos\theta' + v)}$$

Aberration formula

set $\theta' = \pi/2$ shoot up in prime frame

$$\tan\theta = \frac{c}{\gamma v}$$

$$\sin\theta = \frac{1}{\gamma}$$



$$v \approx c \;\longrightarrow\; \sin\theta \sim \theta \sim \frac{1}{\gamma} \ll 1$$

relativistic beaming