$P^\mu = m_0 U^\mu$

$U^\mu U_\mu = -c^2$

$P^\mu P_\mu \to \mathcal{E}^2 = m_0^2 c^4 + c^2 |\vec{p}|^2$

$p^\mu = \begin{pmatrix} \mathcal{E}/c \\ \vec{p} \end{pmatrix} = \begin{pmatrix} \hbar\omega/c \\ \hbar\vec{k} \end{pmatrix}$

define 4 acceleration : $a^\mu = \frac{dU^\mu}{d\tau}$

$p^\mu p_\mu = 0$ $\qquad F^\mu = m_0 a^\mu = \frac{dP^\mu}{d\tau}$

For EM should be related to Lorentz Force $(F_{\mu\nu} U^\mu)$

$F_{LF} = \frac{e}{c} F^\mu_{\ \nu} U^\nu$

$F^\mu_{\ \nu} = F^{\mu\nu}(\eta_{\alpha\nu}) = \begin{pmatrix} -1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix} F^{\mu\nu}$

$F^{\mu\nu} = \begin{bmatrix} 0 & / & / & / \\ E_x & 0 & / & / \\ E_y & -B_y & 0 & / \\ E_z & B_z & -B_x & 0 \end{bmatrix}$

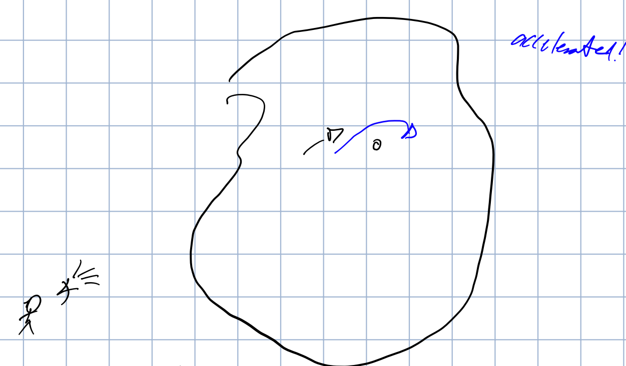
$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$

$\frac{e}{c} F^\mu_{\ \nu} U^\nu = \begin{pmatrix} E & B & 0 \end{pmatrix} \begin{pmatrix} \gamma c \\ \gamma u_x \\ \gamma u_y \\ \gamma u_z \end{pmatrix} = \frac{e\gamma}{c} \begin{pmatrix} \vec{E}\cdot\vec{u} \\ e\gamma(\vec{E}+\vec{u}\times\vec{B}) \end{pmatrix}$

$\frac{1}{m_0} \frac{d}{d\tau}(p^0) = \frac{\gamma}{m_0} \frac{d}{dt}\left(\frac{\mathcal{E}}{c}\right)$

$\frac{d}{dt}(\vec{p}) = e(\vec{E} + \vec{u}\times\vec{B}) \to \frac{dW}{dt} = (\vec{E}\cdot\vec{u})e$

Bremstrahlung



coulomb interactions

what does ♀ see? exactly how clouds of ionized gas heat/cool

assumptions

① Non adiabatic

② No large angle collisions

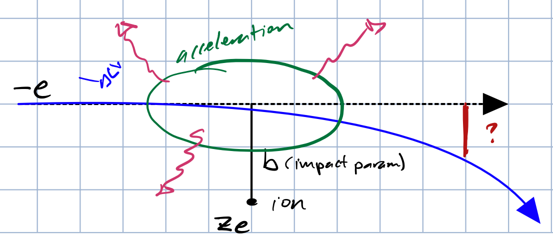
③ Ions are stationary

$n_e \lambda_D^3 \gg 1$ ← most cases $\quad \lambda_D$ - Debye length $\quad \lambda_D^2 = \frac{kT}{4\pi n_e e^2}$

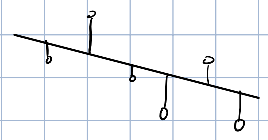
characterizes exponential decay of potential & neutral

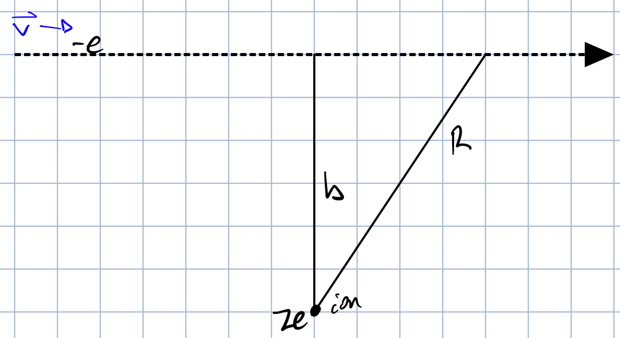
in box $(\lambda_D)^3$ quasineutral, want hella particles to fluidize

3 steps



but we have lots of nucleons. will interact w/ b





use dipole approximation

$\vec{d} = \sum e_i \vec{v}_i$ put origin @ ion's pos^n

$\vec{d} = -e\vec{R}$ $\ddot{\vec{d}} = -e\dot{\vec{v}}$

take Fourier Transform

$$-\omega^2 \hat{d}(\omega) = -\frac{e}{2\pi} \int_{-\infty}^{\infty} \dot{v}\, e^{+i\omega t}\, dt$$

particles interact strongly: $\tau = b/v$ (collision temp)

for $\tau\omega \gg 1$, exp( ) oscillates fast, low effect

for $\tau\omega \ll 1$, exp( ) ≈ 1

To lowest order:

$$\hat{d}(\omega) = \begin{cases} \frac{e}{2\pi\omega} \Delta V & \text{for } \tau\omega \ll 1 \\ 0 & \text{for } \tau\omega \gg 1 \end{cases}$$

ΔV is change in velocity due to collision

$$\frac{dW}{d\omega} = \begin{cases} \frac{2e^2}{3\pi c^2} |\Delta V|^2 & \omega\tau \ll 1 \\ 0 & \omega\tau \gg 1 \end{cases}$$

Assume ΔV is in transverse direction only

integrate transverse component of acceleration

$$\Delta V = \frac{Ze^2}{m}\int_{-\infty}^{\infty} \frac{b \cdot dt}{(b^2+v^2t^2)^{3/2}} \approx \int \frac{1}{b^2}$$

$$\Delta V = \frac{2Ze^2}{mbV}$$

Single collision & small angle scattering gives

$$\frac{dW(b)}{d\omega} = \begin{cases} \frac{8Z^2e^6}{3\pi c^2 m^2 v^2 b^2} & b \ll V/\omega \\ 0 & b \gg V/\omega \end{cases}$$

Suppose we have plasma

n_i – ion density (fixed speed V)

n_e – electron density

electron flux incident on ion $n_e V$

Area element around single ion? $2\pi b\, db$

emission /time/volume/frequency

$$\frac{dW}{dt\,dV\,d\omega} = n_e n_i 2\pi v \int_{b=b_{min}}^{b_{max}} dW(b)\, b\, db = (\text{constants}) \cdot \ln\left(\frac{b_{max}}{b_{min}}\right)$$

what are b_{min} & b_{max}?? ? ¿¿ ¿?

b_{max}: number of order $\frac{V}{\omega}$. take $\frac{V}{\omega} = b_{max}$

b_{min}: exclude particles getting too close (keep small angle approximation)

① small approximation not valid? $b_{min}^{(1)} = \frac{4Ze^2}{\pi m V}$ $\Delta V \sim V$

② QM starts taking place

$\Delta x = b$ $\Delta p = mV$ $\rightarrow$ $\Delta x \Delta p \geq \hbar/2$

$$b_{min}^{(2)} = \frac{\hbar}{mV}$$

$\frac{1}{2}mv^2 << Z^2 \left(\frac{me^4}{2\hbar^2} \right)$

$\ln\left(\frac{b_{max}}{b_{min}}\right) \Rightarrow g(v, \omega)$ Gaunt Factor

everything else stays