additive, Lorentz Transformation

keep Minkowski norm invariant $\begin{pmatrix} \cosh\chi & -\sinh\chi \\ -\sinh\chi & \cosh\chi \end{pmatrix}$ $\tanh\chi = \beta$

$$\tanh(2\chi) = \frac{2\tanh(\chi)}{1+\tanh^2(\chi)}$$

Lorentz group is additive?

story so far

4 - vectors

$$J^\mu = \begin{bmatrix} \rho c \\ \vec{J} \end{bmatrix}$$ contravariant

$$A^\mu = \begin{bmatrix} \phi \\ \vec{A} \end{bmatrix}$$

require $A^\mu_{\ ,\mu} = 0$ $\xrightarrow{\text{4 divergence is zero}}$ Lorentz gauge

(covariant) $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ (Faraday Tensor)

big boi, holds all E/M

$$F_{\mu\nu}^{\ \ ,\nu} = \frac{4\pi}{c} J_\mu$$ gradient wrt ν

$$F_{\mu\nu,\sigma} + F_{\sigma\nu,\mu} + F_{\nu\sigma,\mu} = 0$$ kinda like curl

$$F_{\mu\nu}^{\ \ ,\mu,\nu} = \frac{4\pi}{c} J_\nu^{\ ,\nu} = 0$$ charge conservation

$$F'_{\mu\nu} = \tilde{\Lambda}_\nu^{\ \alpha} \tilde{\Lambda}_\mu^{\ \beta} F_{\alpha\beta}$$ transformation of Faraday tensor

$\tilde{\Lambda}$: just matrices

why 2?: transform f^n & transform derivative

$$\Lambda^\alpha_{\ \beta} \tilde{\Lambda}^\beta_{\ \gamma} = \delta^\alpha_\gamma$$

when sitting @ rest & want to see boosted prime frame

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel} \qquad \vec{B}'_{\parallel} = \vec{B}_{\parallel}$$

$$\vec{E}'_{\perp} = \gamma(\vec{E}_\perp + \vec{\beta} \times \vec{B}) \qquad \vec{B}'_{\perp} = \gamma(\vec{B}_\perp - \vec{\beta} \times \vec{E})$$

Algebra time for $\vec{E}$ & ϕ in retarded time

Look @ transformation of a single charge in rest frame

$$E_x' = q\frac{x'}{r'^3} \qquad B_x' = 0$$

$$E_y' = q\frac{y'}{r'^3} \qquad B_y' = 0$$

$$E_z' = q\frac{z'}{r'^3} \qquad B_z' = 0$$

$$r'^3 = (x'^2 + y'^2 + z'^2)^{\frac{3}{2}}$$

transform to moving frame

$$E_x = q\,\frac{x'}{r'^3} \qquad B_x = 0$$

$$E_y = q\gamma\,\frac{y'}{r'^3} \qquad B_y = -q\gamma\beta\,\frac{z'}{r'^3}$$

$$E_z = q\gamma\,\frac{z'}{r'^3} \qquad B_z = q\gamma\beta\,\frac{y'}{r'^3}$$

Now, transform to coord. system

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

$$E_x = \frac{q\gamma}{r^3}(x - vt) \qquad B_x = 0$$

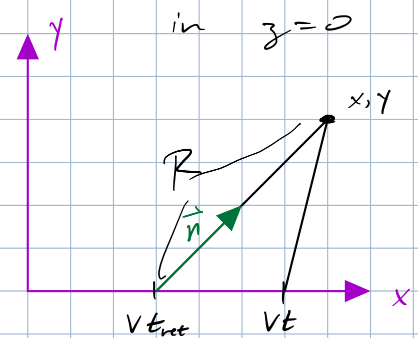
$$E_y = q\gamma\,\frac{y}{r^3} \qquad B_y = -q\gamma\beta\,\frac{z}{r^3}$$

$$E_z = q\gamma\,\frac{z}{r^3} \qquad B_z = q\gamma\beta\,\frac{y}{r^3}$$

$$r^3 = \left(\gamma^2(x-vt)^2 + y^2 + z^2\right)^{3/2}$$

Express in terms of retarded time position

in $z = 0$



$$t_{ret} = t - R/c$$

$$R^2 = y^2 + (x - v t_{ret})^2$$
$$= y^2 + \left(x - vt + \frac{vR}{c}\right)^2$$

quadratic for R^2, solve for R^2

$$R^2 = \gamma^2\beta\,\bar{x} + \gamma\left(y^2 + \gamma^2\bar{x}^2\right)^{1/2} \qquad \bar{x} = x - vt$$

as before...

$$\hat{n} = y\,\hat{y} + \left(x - vt + \frac{vR}{c}\right)\hat{x}$$

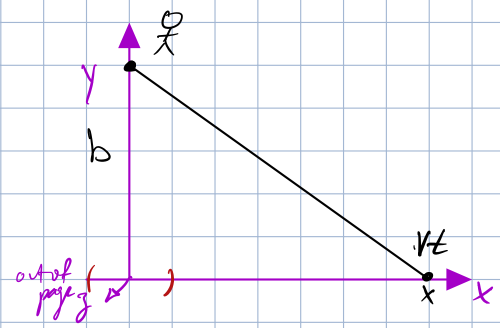
$$k = 1 - \vec{n}\cdot\vec{\beta} \qquad \beta = v/c$$

$$= \frac{(y^2 + \gamma^2\bar{x}^2)^{1/2}}{\gamma R}$$

$$\frac{1}{R^2 k^3} = \frac{\gamma^3 R}{(y^2 + \gamma^2\bar{x}^2)^{3/2}}$$

$$\vec{E} = q\left[\frac{(\vec{n} - \vec{\beta})(1 - \beta^2)}{k^3 R^2}\right]$$

Apply transformation to highly relativistic moving charge



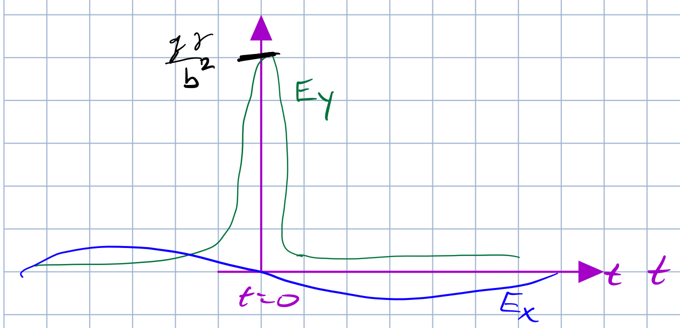
$$E_x = -q\frac{v\gamma t}{(\gamma^2 v^2 t^2 + b^2)^{3/2}} \to r^2 \qquad B_x = 0$$

$$E_y = q\frac{v\gamma t}{(\gamma^2 v^2 t^2 + b^2)^{3/2}} \qquad B_y = 0$$

$$E_z = 0 \qquad B_z = \beta E_y$$

$\beta \approx 1$, can only see something from $t \to 0$

denominator r is small only for $|t| \approx \frac{b}{\gamma v}$



Relativistic Mechanics

define 4 momentum: $p^\mu \equiv m_0 v^\mu$ ← 4 velocity; m_0 ← rest mass!

consider 0^{th} component: cp^0 for $\beta \ll 1$

$$cp^0 = c(c\gamma_u m_0) = m_0 c^2 \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

evaluate γ w/ $v = u$
$$= m_0 c^2 \left(1 + \frac{1}{2}\frac{v^2}{c^2} + \dots\right)$$

$$cp^0 = m_0 c^2 + \frac{1}{2} m_0 v^2 + \dots$$

E^0 energy! $E = mc^2$

rest energy

LMAO: moving mass???
helps transform force?
mass is MASS
doesn't change if moving

$$\vec{p} = \gamma_u m_0 \vec{u}$$

$$p^0 = E/c$$

$$p^\mu = \begin{pmatrix} E/c \\ \vec{p} \end{pmatrix}$$

$U^\mu U_\mu = -c^2$ length of 4 velocity

$$p^\mu p_\mu = -\frac{E}{c}^2 + |\vec{p}|^2 = E^2 = m_0{}^2 c^4 + c^2|\vec{p}|^2$$

4 velocity always $\perp$ to 4 acceleration