

$$x' = -ct, x, y, z \quad \text{contravariant}$$

$$\text{want geometry: metric!} \quad \eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$$

not positive definite

$$x_\mu = ct, x, y, z$$

$$x^\mu x_\mu = x^\mu g_{\mu\nu} \eta^{\nu\mu}$$

invent linear transformation

$$x'^\mu = \lambda^\sigma_\mu x^\sigma$$

$\overset{\uparrow}{\text{Transformation}}$

$$\begin{aligned} x'^\mu x_\mu &= x'^\mu x^\nu \eta_{\nu\mu} \quad \text{to leave } S^2 \text{ invariant} \\ &= x'^\mu x^\nu \eta_{\nu\mu} \quad \text{defn of Lorentz trans.} \\ &= x^\sigma \lambda^\mu_\sigma x^\nu \lambda^\nu_\mu \eta_{\mu\nu} \quad \eta = 1/\lambda^2 \\ &\quad \det(\lambda) = 1 \end{aligned}$$

assume  $\lambda^0 \geq 1$

leave time alone

$$\det(\lambda) = +1$$

$$\left( \begin{smallmatrix} 1 & & \\ & \sqrt{R} & \\ & & 1 \end{smallmatrix} \right) \quad \text{RE O}(3)$$

boost in  $x$ :

$$\lambda^\mu_\nu = \begin{pmatrix} \gamma - \beta\gamma & & \\ \beta\gamma & \gamma & \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{aligned} \beta &= \frac{\nu}{c} \\ \gamma &= \sqrt{1 - \beta^2} \end{aligned}$$

$$x_\mu = \eta_{\mu\nu} x^\nu$$

for covariant

$$x'_\mu = \tilde{\lambda}_\mu^\sigma x_\sigma$$

$$x'^\nu \cdot \eta_{\nu\mu} = \tilde{\lambda}^\nu \lambda^\mu \delta^\sigma_\nu$$

$$\tilde{\lambda}_\nu^\sigma \lambda^\mu_\tau = \delta_\nu^\mu$$

$$\eta_{\mu\sigma} \eta^{\sigma\nu} = \delta_\mu^\nu$$

$$dt = \gamma d\tau$$

$$U^\mu = \frac{d x^\mu}{d\tau} = \left( \frac{c \gamma_u}{\gamma v} \right) \quad \vec{v} = \frac{d \vec{x}}{d\tau}$$

$$\frac{d}{d\tau}(c \cdot \vec{v}_u)$$

$$\frac{d}{d\tau}(v_u \cdot v_x)$$

$$\frac{d}{d\tau}(v_u \cdot v_y)$$

$$\frac{d}{d\tau}(v_u v_z)$$

Then apply the "boost"

$$u^{\mu} = \gamma \left( \begin{smallmatrix} c \\ \vec{u} \end{smallmatrix} \right)$$

$$k^\mu = \left( \begin{smallmatrix} \omega \\ \vec{k} \end{smallmatrix} \right)$$

physics laws have to be Lorentz invariant

$$\vec{k}_0 \vec{x} - \omega t = k_\mu x^\mu$$

$$k^\mu = \left( \begin{smallmatrix} \omega \\ \vec{k} \end{smallmatrix} \right)$$

$$\begin{aligned} k'^0 &= \gamma(k^0 - \beta k^1) \\ k'^1 &= \gamma(-\beta k^0 + k^1) \\ k'^2 &= k^2 \\ k'^3 &= k^3 \end{aligned}$$

$$k' = \frac{\omega}{c} \cos \theta$$

$$\omega' = \omega \gamma (1 - \frac{v}{c} \cos \theta)$$

$\frac{\partial}{\partial x^\mu}$  scalar  $\rightarrow$  covariant vector

$$\frac{\partial}{\partial x^\mu}(\lambda) = \lambda_{,\mu} \text{ comma?}$$

$$\frac{\partial}{\partial x^\mu}(x^\nu) = \delta_\mu^\nu$$

Back to EM! Woo!

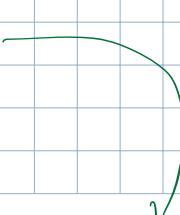
charge is conserved!!!

$$\frac{\partial}{\partial t}(\rho) + \vec{\nabla} \cdot \vec{j} = 0$$

$$j^\mu = \left( \begin{smallmatrix} \rho c \\ \vec{j} \end{smallmatrix} \right) \quad j_{,\mu}^\mu = 0$$

$$\vec{j} = \left( \begin{smallmatrix} j_x \\ j_y \\ j_z \end{smallmatrix} \right)$$

"4 divergence" comma



$$(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial z^2}) \phi = -4\pi e$$

$$(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) \vec{A} = -\frac{4\pi}{c} \vec{j}$$

$$\nabla^2 \equiv \nabla \cdot \nabla$$

$$A^\mu = \begin{pmatrix} \phi \\ \vec{A} \end{pmatrix}$$

$$A^{\mu,\nu} = -4\pi J^\mu$$

$\uparrow$   
divergence of gradient

$$A_{,\mu}^\mu = 0 \rightarrow \frac{1}{c} \frac{\partial}{\partial z} (\phi) + \nabla \cdot \vec{A} = 0 \quad (\text{Lorentz gauge})$$

#### 4 Vector representation

must contain derivatives of  $A^\mu$

Faraday Tensor:  $F_{\mu\nu}$

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} = A_{[\nu,\mu]}$$

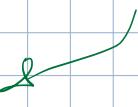
$$F_{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

antisymmetric

$F_{\mu\nu}^{\prime,\nu} = \frac{4\pi}{c} J_\mu$   
holds 2 inhomogeneous  
Maxwell's eq's

divergence ???

$$F_{\mu\nu}^{\prime,\mu\nu} = \frac{4\pi}{c} J_{\nu}^{\prime,\nu} = 0$$



charge conservation

Geometric Maxwell's eq's

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial}{\partial t} (\vec{B})$$

$$F_{\mu\nu,\sigma} + F_{\nu\mu,\sigma} + F_{\nu\sigma,\mu} = 0$$

Person moving says my (I'm not moving)  $\vec{E}$  is actually  $\vec{B}$  & my stationary charges look like moving charges

so  $\vec{B} \neq \vec{E}$  can't really be trusted, but Faraday tensor is invariant

$$F'_{\mu\nu} = \tilde{x}_\mu^\alpha \tilde{x}_\nu^\beta F_{\alpha\beta}$$

$$\vec{E}'_{||} = \vec{E}_{||}$$

want spatial transformations

$$\vec{B}'_{||} = \vec{B}_{||}$$

$$\vec{E}'_\perp = \gamma(\vec{E}_\perp + \vec{\beta} \times \vec{B}_\perp)$$

$$\beta = \frac{v}{c}$$

$$\vec{B}'_\perp = \gamma(\vec{B}_\perp - \vec{\beta} \times \vec{E}_\perp)$$