$x^\mu = -ct, x, y, z$ contravariant

want geometry: metric! $\eta^{\mu\nu} = \mathrm{diag}(-1,1,1,1)$

not positive definite

$x_\mu = ct, x, y, z$

$$x^\mu x_\mu = x^\mu \mu\sigma\ \eta_{\sigma\mu}$$

invent linear transformation

$$x'^\mu = \Lambda_\mu^{\ \sigma}\ x^\mu$$

↑ transformation

$$x^\mu x_\mu = x^\mu x^\nu \eta_{\nu\mu}$$

$$= x'^\mu\ x'^\nu\ \eta_{\nu\mu}$$

$$= x^\sigma \Lambda_\sigma^{\ \mu}\ x^\nu \Lambda_\nu^{\ \nu}\ \eta_{\nu\mu}$$

b/c leave s^2 invariant

def^n of Lorentz trans.

$\eta = \Lambda \Lambda_\eta$

$\det^2(\Lambda) = 1$

$\det(\Lambda) = +1$

assume $\Lambda_0^{\ 0} \geq 1$ leave time alone

$$\begin{pmatrix} 1 & \\ & R \end{pmatrix} \quad R \in O(3)$$

boost in x:

$$\Lambda_\nu^{\ \mu} = \begin{pmatrix} \gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & 0 \\ & & 1 & \\ & 0 & & 1 \end{pmatrix}$$

$\beta = \frac{v}{c}$

$\gamma = \left(\sqrt{1-\beta^2}\right)^{-1}$

$$x_\mu = \eta_{\mu\nu}\ x^\nu$$

for covariant

$$x'_\mu = \tilde{\Lambda}_\mu^{\ \sigma}\ x_\sigma$$

$$x'^\nu \cdot \eta_{\mu\nu} = \tilde{\Lambda}\Lambda\ x^\sigma\ \mu^{\sigma\nu}$$

$$\tilde{\Lambda}_\nu^{\ \sigma}\ \Lambda_\tau^{\ \mu} = \delta^\mu_{\ \nu} \qquad \eta_{\mu\sigma}\eta^{\sigma\nu} = \delta_\mu^\nu$$

$$dt = \gamma\, d\tau$$

$$U^\mu = \frac{dx^\mu}{d\tau} = \begin{pmatrix} c\gamma_u \\ \vec{v}\gamma_u \end{pmatrix} \qquad \vec{v} = \frac{d\vec{x}}{dt}$$

$$\frac{d}{d\tau}(c \cdot \gamma_u)$$

$$\frac{d}{d\tau}(\gamma_u \cdot v_x)$$

$$\frac{d}{d\tau}(\gamma_u \cdot v_y)$$

$$\frac{d}{d\tau}(\gamma_u v_z)$$

then apply the "boost"

$$u'^\mu = \gamma' \begin{pmatrix} c \\ \vec{u}' \end{pmatrix}$$

$$K^\mu = \begin{pmatrix} \omega \\ \vec{k} \end{pmatrix}$$

physics laws have to be Lorentz invariant

$$\vec{k} \circ \vec{x} - \omega t = k_\mu x^\mu$$

$$k^\mu = \begin{pmatrix} \omega/c \\ \vec{k} \end{pmatrix}$$

$$k'^0 = \gamma(k^0 - \beta k^1)$$
$$k'^1 = \gamma(-\beta k^0 + k^1)$$
$$k'^2 = k^2$$
$$k'^3 = k^3$$

$$k^1 = \frac{\omega}{c}\cos\theta$$

$$\omega' \equiv \omega\gamma\left(1 - \frac{v}{c}\cos\theta\right)$$

$\frac{\partial}{\partial x^\mu}$ scalar $\rightarrow$ covariant vector

$$\frac{\partial}{\partial x^\mu}(\lambda) = \lambda_{,\mu}$$

comma?

wrt

$$\frac{\partial}{\partial x^\mu}(x^\nu) = \delta^\nu_\mu$$

Back to EM! woo!

charge is conserved!!! $\frac{\partial}{\partial t}(\rho) + \vec{\nabla} \cdot \vec{J} = 0$

$$J^\mu = \begin{pmatrix} \rho c \\ \vec{J} \end{pmatrix} \qquad J^\mu{}_{,\mu} = 0$$

$$\vec{J} = \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}$$

"4 divergence" comma

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = -4\pi\rho$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial}{\partial t^2}\right)\vec{A} = -\frac{4\pi}{c}\vec{J}$$

$$\nabla^2 \equiv \nabla \cdot \nabla$$

$$A^\mu = \begin{pmatrix}\Phi \\ \vec{A}\end{pmatrix}$$

$$A^{\mu,\nu}{}_{,\nu} = -4\pi J^\mu$$

divergence of gradient

$$A^\mu{}_{,\mu} = 0 \rightarrow \frac{1}{c}\frac{\partial}{\partial t}(\phi) + \nabla \vec{A} = 0$$ (Lorentz gauge)

4 Vector representation

must contain derivatives of A^μ

Faraday Tensor: $F_{\mu\nu}$

$$F_{\mu\nu} \equiv A_{\nu,\mu} - A_{\mu,\nu} = A_{[\nu,\mu]}$$

$$F_{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{bmatrix}$$

antisymmetric

$$F_{\mu\nu}{}^{,\nu} = \frac{4\pi}{c} J_\mu$$

holds 2 inhomogeneous Maxwell's eq'ns

divergence???

$$F_{\mu\nu}{}^{,\mu\nu} = \frac{4\pi}{c}\underline{J_\nu{}^{,\nu} = 0}$$

charge conservation

Geometric Maxwell's eqⁿ's

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}(\vec{B})$$

$$F_{\mu\nu,\sigma} + F_{\sigma\mu,\nu} + F_{\nu\sigma,\mu} = 0$$

person moving says my (I'm not moving) $\vec{E}$ is actually $\vec{B}$ b/c my stationary charges look like moving charges

so $\vec{B}$ & $\vec{E}$ can't really be trusted, but Faraday tensor is invariant

$$F'_{\mu\nu} = \tilde{\Lambda}_{\mu}^{\alpha}\,\tilde{\Lambda}_{\nu}^{\beta}\,F_{\alpha\beta}$$

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel}$$

want spatial transformations

$$\vec{B}'_{\parallel} = \vec{B}_{\parallel}$$

$$\vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \vec{\beta}\times\vec{B}_{\perp})$$

$$\beta = \frac{v}{c}$$

$$\vec{B}'_{\perp} = \gamma(\vec{B}_{\perp} - \vec{\beta}\times\vec{E}_{\perp})$$