Total emitted power!

$$\frac{dW}{dt} = \frac{dW'}{dt'} \qquad dW = \gamma dW' \qquad dt = \gamma dt'$$

$$P' = \frac{2q^2}{3c^3} \cdot |\vec{a}'|^2 = \frac{2q^2}{3c^3} \cdot a^\mu a_\mu \qquad \vec{a} = \frac{d}{d\tau}(\vec{v})$$

power, not momentum

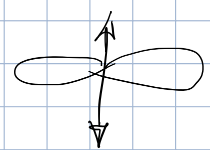
$$a_\mu = \frac{dU^\mu}{d\tau}$$

$$|\vec{a}'|^2 = a^\mu a_\mu \qquad U^\mu a_\mu = \begin{pmatrix} c \\ 0 \\ 0 \\ 0 \end{pmatrix} a_\mu = \begin{pmatrix} 0 \\ \vec{a} \end{pmatrix}$$

K' rest frame of emitter

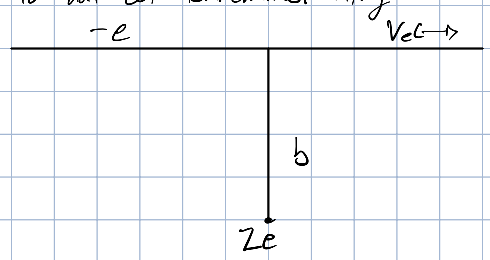
$$a'_{\parallel} = \gamma^3 a_{\parallel}$$

$$a'_{\perp} = \gamma^2 a_{\perp}$$



$$P = \frac{2q^2}{3c^3} \cdot (a_{\parallel}'^2 + a_{\perp}'^2) = \frac{2q^2}{3c^3} \gamma^4 (\gamma^2 a_{\parallel}^2 + a_{\perp}^2)$$

back to dat boi Bhremmstrauling



@ low energies: $h\nu \ll m_e c^2$

Thomson scattering by pulse

@ high energies: $h\nu \gg m_e c^2$

Compton scattering

$$E_y(t) = \frac{q\gamma b}{(\gamma^2 v^2 t^2 + b^2)^{3/2}}$$

$$\hat{E}(\omega) = \frac{1}{2\pi} \int E_y(t)\, e^{i\omega t}\, dt$$

$$\Rightarrow \hat{E}(\omega) = \left(\frac{q}{\pi b v}\right) \cdot \left(\frac{b\omega}{\gamma v}\right) \cdot \underbrace{K_1\left(\frac{b\omega}{\gamma v}\right)}$$

→ Modified Bessel f^n of 2^{nd} kind of order 1

$$\frac{dW}{dA\, d\omega} = c \cdot |\hat{E}(\omega)|^2 = \frac{q^2 c}{\pi^2 b^2 v^2} \cdot \left(\frac{b\omega}{\gamma v}\right)^2 \cdot K_1^2\left(\frac{b\omega}{\gamma v}\right)$$

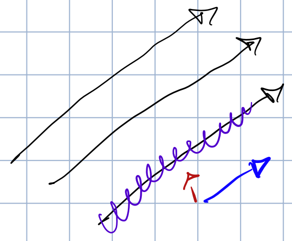
eqn 4.72

for prime, $v \to c$

$$\frac{dW'}{d\omega'} = \sigma_T \cdot \frac{dW'}{dA'\, d\omega'} \qquad \frac{dW}{d\omega'} = \frac{dW}{d\omega} \qquad \omega = \gamma\omega'(1+\beta\cos\theta')$$

$$\omega = \gamma\omega'$$

$$\frac{dW}{d\omega} = \frac{8 z^2 e^6}{3\pi b^2 c^5 m^2} \cdot \left(\frac{b\omega}{\gamma^2 c}\right)^2 \cdot K_1^2\left(\frac{b\omega}{\gamma c}\right)$$



$\vec{B}_0$

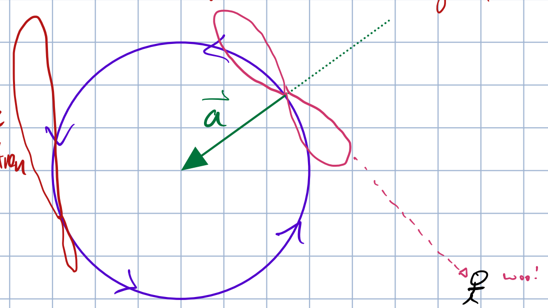
Synchrotron

$\vec{v}$ along $\vec{B}$ stays constant if $\vec{B}$ is constant

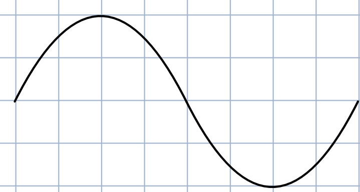
$\vec{v} \perp \vec{B}$ is just hella (sin & cos)'s

relativistic

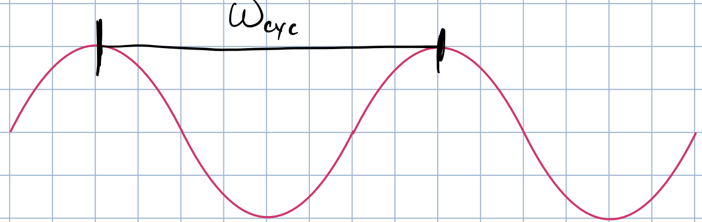
only signal in 1 direction



see nothing :(

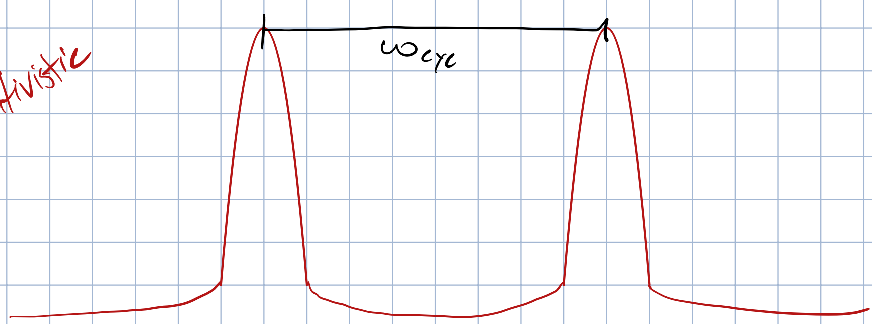


regular



ω_{cyc} - cyclotron frequency

relativistic



picks up γ^3 factor

pitch angle: $\frac{v_\parallel}{v_\perp}$

integrate over all pitch angles & γ's

not usually thermalized

γ is in powerlaw, can rewrite as sum of Maxwellians

$$\frac{d}{dt}(\gamma m \vec{v}) = \frac{q}{c} \vec{v} \times \vec{B}$$

$\vec{E} = 0$ (can do b/c frame of reference)

$$\frac{d}{dt}(\gamma m c^2) = q \vec{v} \cdot \vec{E} = 0$$

$\Rightarrow \gamma$ constant $\Rightarrow |\vec{v}|$ constant, can pull in $\frac{d}{dt}$

$$\gamma m \frac{d}{dt}(\vec{v}) = \frac{q}{c}\vec{v} \times \vec{B}$$

$$\frac{d}{dt}(v_{\parallel}) = 0 \qquad v_{\parallel} \text{ constant}$$

$$\frac{d}{dt}(v_{\perp}) = \frac{q}{\gamma m c} v_{\perp} \times \vec{B} \qquad \text{choose } \vec{B} = (0,0,B_0) \text{ \& } \vec{v} = (v_x, v_y, v_z)$$

$$\frac{d}{dt}\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \frac{q}{m\gamma c}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} v_x \\ v_z \end{pmatrix}$$

$$v_x = v_{\perp}\sin(\omega t + \phi)$$

$$v_y = v_{\perp}\cos(\omega t + \phi)$$

$r_c = \frac{v_{\perp}}{\omega}$ Larmor radius

$\omega = \frac{q B_0}{\gamma m c}$ cyclotron frequency

$\alpha =$ pitch angle

$$v_{\parallel} = v\cos(\alpha)$$

$$v_{\perp} = v\sin(\alpha)$$

get $\vec{a}$ & plug into Larmor's formula $\qquad U_B = \frac{B_0^2}{8\pi}$

$$P_{ower} = \left(\frac{2}{3}\right) r_0^2 \cdot c\gamma^2\beta^2 B_0^2 = \frac{4}{3}\sigma_T c\beta^2\gamma^2 \cdot U_B$$