day 3 of escuela! :)

Extensions of postulates of QM

allow for flexibility of different systems: degeneracy

① Degeneracy: an observable θ s.t. $\theta|\lambda_i\rangle = \lambda_i|\lambda_i\rangle$

state $|\psi\rangle$

prob. of eigenvalue λ_i = $|\langle\lambda_i|\psi\rangle|^2$

what if there are several states w/same eigenvalue

interested in $\underline{\lambda}$ → $\theta|\phi_i\rangle = \lambda|\phi_i\rangle$ $i = 1, 2, \cdots, N$

prob. of measuring λ: $p(\lambda) = \sum_{i=1}^{N} |\langle\phi_i|\psi\rangle|^2$ sum of all ways of getting λ

The state after measurement is

projection operator onto subspace of $\mathcal{H}$ w/eigenvalue

$$\sum_i \langle\phi_i|\psi\rangle\,|\phi_i\rangle = \Big(\sum_i |\phi_i\rangle\langle\phi_i|\Big)\,|\psi\rangle$$

We know $|\psi\rangle$

then need to normalize again

② What do we do when we don't know what state we are in?

lol: IL? QM?

just like in Classical Mechanics $(\vec{x}_i, \vec{p}_i)$ point in phase space

don't know $|\psi\rangle$ analog in QM $|\psi_i\rangle$ - probability of being in this state

$\sum p_i = 1$

different situations

consider observable θ, expectation value in state $|\psi\rangle$: $\langle\psi|\theta|\psi\rangle$

define statistical expectation value: $\langle\theta\rangle_{stat} = \sum_i p_i \langle\psi_i|\theta|\psi_i\rangle$

often formulated w/ density matrix/operator

projection operator onto state $|\psi_i\rangle\langle\psi_i|$

$$\rho = \sum p_i\,|\psi_i\rangle\langle\psi_i|$$

sum of diagonals

$\langle\psi_i|\psi_j\rangle = \delta_{ij}$

can rewrite $\langle\theta\rangle_{stat}$ as: $\langle\theta\rangle_{stat} = Tr_{\mathcal{H}}(\rho\theta) = \sum_j \langle\psi_j|\rho\theta|\psi_j\rangle$

2 ways to write: ① projection ② trace

$$\langle O \rangle_{stat} = \sum_j \langle \psi_j | \left(\sum_i p_i |\psi_i\rangle\langle\psi_i| \right) O | \psi_j \rangle$$

$$= \sum_j \sum_i p_i \underbrace{\langle \psi_j | \psi_i \rangle}_{\delta_{ij}} \langle \psi_i | O | \psi_j \rangle$$

$$= \sum_i p_i \langle \psi_i | O | \psi_i \rangle$$

Nomenclature: if $\rho^2 = \rho$ – pure state ← so far, only 1 possible case
else $\rho^2 \neq \rho$ – mixed state ← more than 1 possible state w/ non zero probabilities

$$\rho^2 = \left(\sum_i p_i |\psi_i\rangle\langle\psi_i| \right) \left(\sum_j p_j |\psi_j\rangle\langle\psi_j| \right)$$

$$= \sum_i \sum_j p_i p_j |\psi_i\rangle \underbrace{\langle \psi_i | \psi_j \rangle}_{\delta_{ij}} \langle \psi_j |$$

$$= \sum_i p_i p_i |\psi_i\rangle\langle\psi_i| = \sum_i p_i^2 |\psi_i\rangle\langle\psi_i|$$

if $\rho^2 = \rho \rightarrow \langle \psi_j | \rho^2 | \psi_j \rangle = \langle \psi_j | \rho | \psi_j \rangle$

$$\langle \psi_j | p_i^2 |\psi_i\rangle\langle\psi_i|\psi_j\rangle = \langle\psi_j| p_i |\psi_i\rangle\langle\psi_i|\psi_j\rangle$$

$$p_i^2 \langle \psi_j | \psi_i \rangle = p_i \langle \psi_j | \psi_i \rangle$$

$$p_i^2 = p_i$$

$$\longrightarrow p_i = \{0, 1\}$$

looking @ condition $\sum_i p_i = 1$, one prob is 1, all others are 0

Apply density matrices & properties w/ entanglement

Alice & Bobby boi

Alice & Bob each w/ spin ½ particles

$\mathcal{H} = V_{1/2} \otimes V_{1/2}$ $\qquad V_A = \mathbb{C}^2 \qquad V_B = \mathbb{C}^2$

$\qquad\qquad\qquad |+\rangle_A \;\; |-\rangle_A \qquad |+\rangle_B \;\; |-\rangle_B$

$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \sigma_3 |\pm\rangle = \pm|\pm\rangle$ or $S_z = \frac{\hbar}{2}\sigma_3$

$\mathcal{H}$ has basis

$|+\rangle_A \otimes |+\rangle_B \equiv |++\rangle$

$|+\rangle_A \otimes |-\rangle_B \equiv |+-\rangle$

$|-\rangle_A \otimes |+\rangle_B \equiv |-+\rangle$

$|-\rangle_A \otimes |-\rangle_B \equiv |--\rangle$

convention for abbreviation

Winter21: state in $\mathcal{H}$ is entangled if can't be written in form $|\chi\rangle_A \otimes |\psi\rangle_B$

example

1. $|\psi\rangle = |+\rangle_A \otimes |+\rangle_B \rightarrow$ unentangled

2. $|\chi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_A \otimes |-\rangle_B - |-\rangle_A \otimes |+\rangle_B\right) \rightarrow$ entangled

there is equivalent def^n w/ density matrices