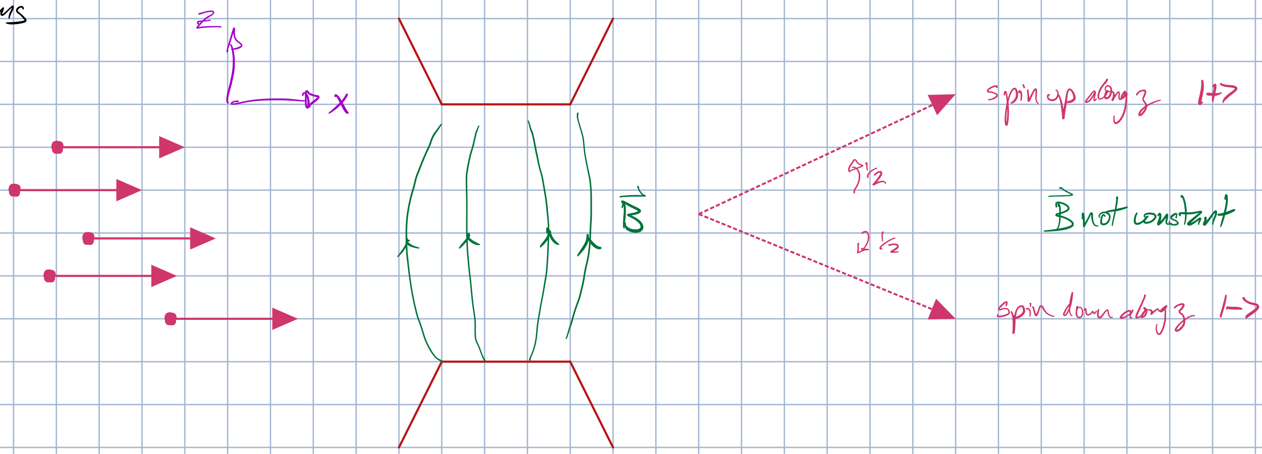
Spin 1/2 – measuring spin 1/2

Stern-Gerlach 1922 — before Heisenberg & Schrödinger's QM papers, before spin 1/2 was known

Silver atoms



$Q_{pe} = 0$ but spin $= 1/2$ → magnetic field

Silver atoms has 47 protons & 47 electrons. 46 of electrons are in a closed shell. Closed shell makes it chemically inert & closed shell w/ $\vec{J}=0$. All spins with the closed shells pair up to cancel each other. But not true for 47th electron b/c it is so weakly bound. 47th electron's spin 1/2 effectively gives Silver spin 1/2

① How does it work?
② What happens when we combine SG devices

$$\text{Energy} = -\vec{\mu}\cdot\vec{B}$$

$$\text{Force}: \vec{F} = -\vec{\nabla}(-\vec{\mu}\cdot\vec{B}) = \vec{\nabla}(\vec{\mu}\cdot\vec{B}) \overset{\text{if } \vec{B} \text{ is constant}}{=} 0$$

$$\vec{B} = B_0\hat{e}_z + \delta B(z\hat{e}_z - x\hat{e}_x)$$

$\vec{\nabla}\cdot\vec{B} = 0$ no divergence

$$H = \frac{\vec{p}^{\,2}}{2m} - \vec{\mu}\cdot B \qquad \vec{\mu} = \gamma S$$

$$H = \frac{\vec{p}^{\,2}}{2m} - \gamma\,\delta B\underbrace{(-xS_x + zS_z)}_{\text{force}} - \underbrace{\gamma B_0 S_z}_{\text{precesses}}$$

motion in x-direction is irrelevant, SG measures S_z

$$\text{if } H = -\gamma\,\delta B\, z S_z = -\gamma\,\delta B\, z\,\frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \qquad |a(0)|^2 + |b(0)|^2 = 1$$

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

$$\begin{pmatrix} \partial a/\partial t \\ \partial b/\partial t \end{pmatrix} = \frac{-\gamma S B_z \hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

Solution: $\psi(t) = a(0)\, e^{i\gamma S B_z t/2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b(0)\, e^{-i\gamma S B_z t/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

after time T

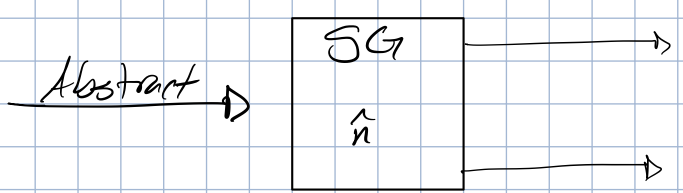
$$\to \psi = \underbrace{a(0)\, e^{i\gamma S B_z T/2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}} + b(0)\, \underbrace{e^{-i\gamma S B_z T/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$

remember, momentum eigenstate: $e^{i\vec{p}\cdot\vec{x}/\hbar}$

$$p_z = \frac{\gamma S B T \hbar}{2} \qquad\qquad p_z = -\frac{\gamma S B T \hbar}{2}$$

if $p_z > 0$, end up measuring $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\nearrow$

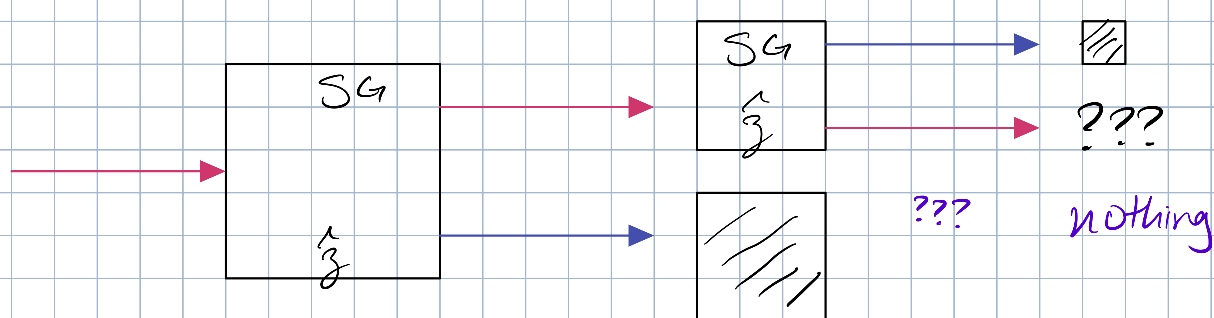
if $p_z < 0$, end up measuring $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\searrow$



spin up along $\hat{n}$ $\qquad \hat{n}\cdot\vec{S}\,|\text{state}\rangle = +\frac{\hbar}{2}\,|\text{state}\rangle$

spin down along $\hat{n}$ $\qquad \hat{n}\cdot\vec{S}\,|\text{state}\rangle = -\frac{\hbar}{2}\,|\text{state}\rangle$

1



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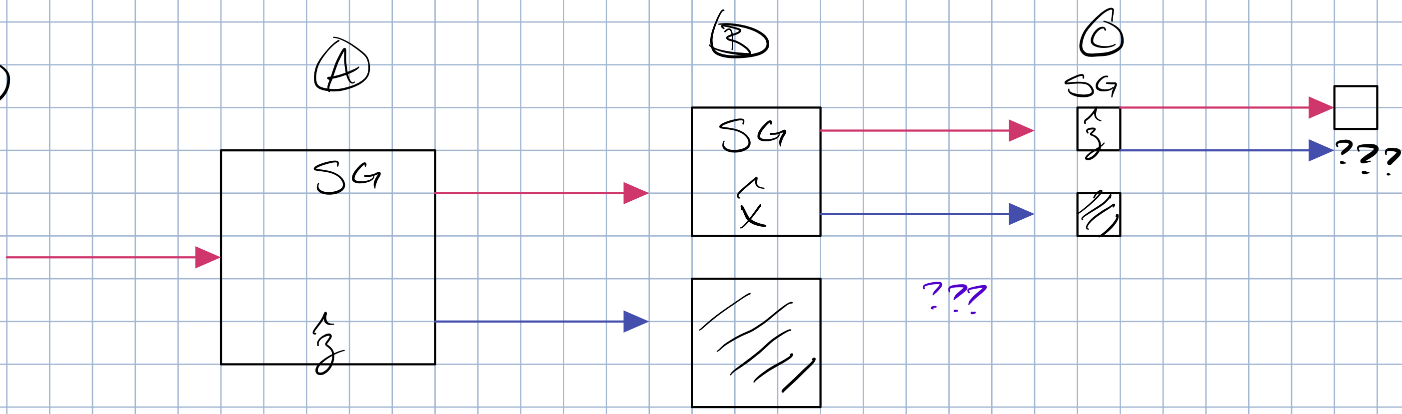


A $\to$ $|+,z\rangle$

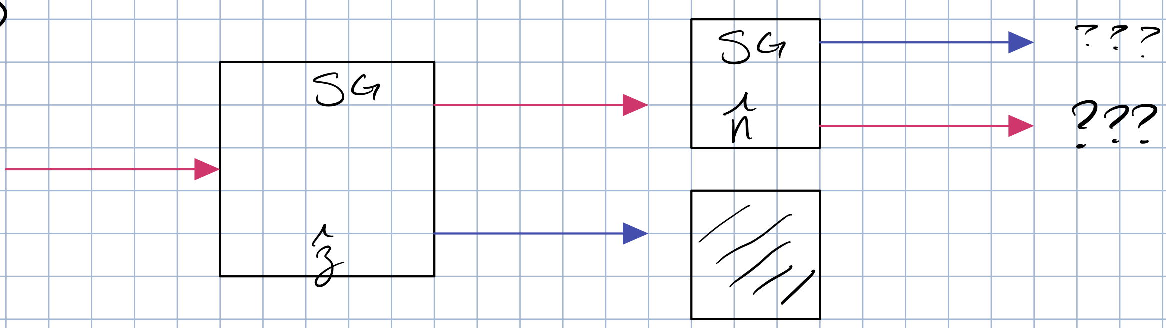
$$|\pm,x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm 1\end{pmatrix} = \frac{1}{\sqrt{2}}\left(|+,z\rangle \pm |-,z\rangle\right)$$

B $\to$ $|+,z\rangle = \frac{1}{\sqrt{2}}\Big(\underbrace{|+,x\rangle}_{\text{spin up is 1/2 probability}} + \overset{\text{absorbed}}{\cancel{|-,x\rangle}}\Big)$

C $\to$ $|+,x\rangle = \frac{1}{\sqrt{2}}\Big(\underbrace{|+,z\rangle}_{\text{spin up is 1/2}} + \overset{\text{absorbed}}{\cancel{|-,z\rangle}}\Big)$

$$\hat{n} = \hat{e}_z \cos\theta + \hat{e}_y \sin\theta$$

3



How to use quantum entanglement to correct quantum errors

bits: 0, 1

qubits: $|\psi\rangle = a\overset{|+\rangle}{|0\rangle} + b\overset{|-\rangle}{|1\rangle}$

S^2 worth of states
2 sphere

N qubits $\quad \mathcal{H} = \underbrace{(\mathbb{C}^2) \otimes (\mathbb{C}^2) \otimes (\mathbb{C}^2) \otimes \ldots \otimes (\mathbb{C}^2)}_{N \text{ copies}}$

$|0\rangle \otimes |1\rangle \otimes |1\rangle \otimes \ldots \otimes |0\rangle \to |011 \ldots 0\rangle$

Time evolution is unitary : $|\psi(t)\rangle = \underbrace{e^{-iHt/\hbar}}_{\to U} |\psi(0)\rangle$ $\qquad U^\dagger U = U U^\dagger = \mathbb{1}$

$i\hbar \frac{\partial \psi}{\partial t} = H\psi$

U - 2x2 unitary matrix

$U_{ab} = \langle a|U|b\rangle \qquad a = 0,1 \quad b = 0,1$

all 2x2 unitary matrices is group $U(2)$

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_x$

$Y = i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \sigma_y$

$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \sigma_z$

$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z)$ $\qquad$ Hadamard gate