plan Office Hours before midterm

Midterm: Friday Oct. 29th

Linear Algebra reminder:

$|\psi\rangle \in V$

$\langle\psi| \in V^*$ $\qquad\qquad \langle\psi| : V \to V^*$

$\langle\psi| \left(|\chi\rangle\right) = \langle\psi|\chi\rangle$

say $\langle\psi|\psi\rangle = 1$

Projection operator: $\quad P_\psi = |\psi\rangle\langle\psi| : \quad V \to V$

$$P_\psi |\chi\rangle = \underbrace{|\psi\rangle}_{\to V} \underbrace{\langle\psi|\chi\rangle}_{\to \mathbb{C}} \in V$$

Tensor Product

on $V_A \otimes V_B$

take $|\psi\rangle \in V_A \otimes V_B$ $\qquad\qquad |\psi\rangle_A \in V_A$

$\langle\psi| : V_A \otimes V_B \to \mathbb{C}$

$|\psi\rangle\langle\psi| : V_A \otimes V_B \to V_A \otimes V_B$

${}_A\langle\psi| : V_A \otimes V_B \to V_B$

$${}_A\langle\psi| \left(|\chi\rangle_A \otimes |\phi\rangle_B \right) = \underbrace{{}_A\langle\psi|\chi\rangle_A}_{\to \mathbb{C}} \cdot \underbrace{|\phi\rangle_B}_{\to V_B} \in V_B$$

onto density matrices!

A complication: $|\psi\rangle \in V_A \otimes V_B$

$\rho_A = \mathrm{Tr}_{V_B}(|\psi\rangle\langle\psi|)$ may not be diagonal

$\rightarrow$ need to diagonalize to find probabilities for ρ_A

$$|\phi\rangle = \frac{1}{4}\left(|++\rangle + \sqrt{3}\,|+-\rangle + \sqrt{3}\,|-+\rangle + 3\,|--\rangle\right)$$

$$\rho_B = {}_A\langle +|\phi\rangle\langle\phi|+\rangle_A + {}_A\langle -|\phi\rangle\langle\phi|-\rangle_A$$

$$= \frac{1+\sqrt{3}}{4}\begin{pmatrix}\sqrt{3} & 3\\ 3 & 3\sqrt{3}\end{pmatrix} \qquad \det(\rho_B) = 0$$

"don't freak out, diagonalize!"

density operators are Hermitian, Trace 1, & positive operators

Starting def^n: $*\ \rho_A = \sum_i p_i\,|\psi_i\rangle\langle\psi_i|$ for bases $|\psi_i\rangle$

another way to define

$i \to i$ $a \to j$

$b \to k$ $k \to p$ $l \to q$

$\hookrightarrow$ his var $\to$ my var

$|\psi\rangle \in V_A \otimes V_B$

take $|i\rangle_A$ & $|j\rangle_B$ ON bases for $V_A \otimes V_B$

$|\psi\rangle = \sum_{i,j} C_{ij}\,|i\rangle_A \otimes |j\rangle_B$

also say $|\psi\rangle$ can be norm'd $\rightarrow$ $\langle\psi|\psi\rangle = 1 \rightarrow \sum_{i,j}|C_{ij}|^2 = 1$

$$\rho_A = \mathrm{Tr}_{V_B}(|\psi\rangle\langle\psi|)$$

$$= \sum_k {}_B\langle k|\psi\rangle\langle\psi|k\rangle_B$$

$$= \sum_k {}_B\langle k|\underbrace{\Big(\sum_{i,j} c_{ij}\,|i\rangle_A \otimes |j\rangle_B\Big)}_{|\psi\rangle}\overbrace{\Big(\sum_{pq} c_{pq}^*\,{}_A\langle p| \otimes {}_B\langle q|\Big)}^{\langle\psi|}|k\rangle_B$$

δ_{kj} δ_{kq}

$$= \sum_k \sum_i c_{i,k}\,|i\rangle_A \sum_p c_{pk}^*\,{}_A\langle p|$$

$$\rho_A = \sum_k \sum_i \sum_p c_{i,k}\,c_{p,k}^*\,|i\rangle_A\,{}_A\langle p|$$

$$Tr(\rho_A) = \sum_\gamma {}_A\langle\gamma|\rho_A|\gamma\rangle_A = \sum_\gamma \sum_k |c_{\gamma k}|^2 = 1$$

can check positivity: ${}_A\langle\phi|\rho_A|\phi\rangle_A \geq 0$

$\mathcal{H} \in \mathbb{C}^2$

$$\rho = 0.999\, |+\rangle\langle+| + 0.001\, |-\rangle\langle-|$$

not pure, but close. what does close mean?

is there way to measure how close ρ is to being pure state?

Von Neumann entropy

$$S = -(k_B)\, Tr(\rho) \cdot \ln(\rho)$$???!

if A is a matrix, $\ln(A)$ is matrix s.t. $e^{\ln(A)} = A$

always bases s.t.: $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

$$\rho = diag(p_1, p_2, \ldots, p_N)$$

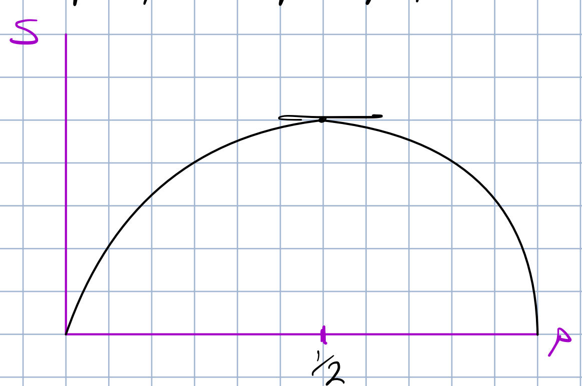
$$\ln(\rho) = diag(\ln(p_1), \ln(p_2), \ldots, p_N)$$

$$S = \sum_i p_i \ln(p_i)$$

$\rho \to U\rho U^\dagger$, S is invariant

also rewrite $\rho = p|+\rangle\langle+| + (1-p)|-\rangle\langle-|$

$$S = -p\ln(p) - (1-p)\ln(p-1)$$



$S = 0$ for $|+\rangle\langle+|$ or $|-\rangle\langle-|$

maximal for $\frac{1}{2}|+\rangle\langle+| + \frac{1}{2}|-\rangle\langle-|$

Time to consider N 2state systems

$\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \ldots\ldots$

two state $\rightarrow$ qubit

$|+\rangle, |-\rangle$ or $|0\rangle, |1\rangle$

$|\psi\rangle = \alpha|+\rangle + \beta|-\rangle \in \mathbb{C}^2$ qubit!

take sequence of bits $\rightarrow$ manipulate $\rightarrow$ algo

Stop & look at $N = 3$

Greenberger - Horne - Zeilenger

GHZ

$|+\rangle_A \otimes |-\rangle_B \otimes |+\rangle_C \otimes |+\rangle_D \otimes \ldots = |+-++\ldots\rangle$

generalization of Bell's Thm

1990

$$|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|{+}{+}{+}\rangle - |{-}{-}{-}\rangle\right)$$