No mo' codes ;)

Vacuum Eq^{n}'s

$\nabla \cdot \vec{B} = 0$ ①

$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$ ③

$\nabla \cdot \vec{E} = 4\pi\rho$ ②

$\nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{j}$ ④

in presence of ρ & $\vec{j}$
cgs units! not MKS

QM systems w/ charged particles interacting w/ classical $\vec{E}$ & $\vec{B}$ fields

$\nabla \cdot \vec{B} = 0$ can be solved w/ $\vec{B} = \nabla \times \vec{A}$ (b/c $\nabla \cdot (\nabla \times \vec{A}) = 0$)

$\longrightarrow \quad \nabla \times \left(\vec{E} + \frac{1}{c}\frac{\partial \vec{A}}{\partial t}\right) = 0$

$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} +$ something w/ zero curl

$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \nabla\phi$

$\vec{B} = \nabla \times \vec{A}$ ①

$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \nabla\phi$ ②

$\vec{A}$ - vector potentials

ϕ - scalar potentials

can plug into ③ & ④ to get wave eq^{n} for $\vec{A}, \phi$ ⑤

$A_\mu = (\phi, \vec{A})$
gauge field

$j_\mu = (\rho, \vec{j})$

$\vec{A}$ & ϕ are not unique.

$\nabla \times (\nabla \cdot f) = 0$

$$\left.\begin{aligned} \vec{A}' &= \vec{A} + \nabla\Lambda(\vec{x}, t) \\ \phi' &= \phi - \frac{1}{c}\frac{\partial}{\partial t}\left(\Lambda(\vec{x}, t)\right) \end{aligned}\right\} \text{ gauge transformation}$$

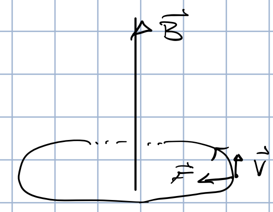
$\vec{E}, \vec{B}$ are same whether use $(\phi, \vec{A})$ or $(\phi', \vec{A}')$

"choose a gauge" - impose condition on $\vec{A}$ & ϕ to remove ambiguity

$\vec{A} = \left(A_x(x,y,z,t),\ A_y(x,y,z,t),\ A_z(x,y,z,t)\right)$

Classically, particle w/ charge q obeys equations of motion (eom)

$$m \frac{d^2\vec{x}}{dt^2} = q\vec{E} + \frac{q}{c}\left(\frac{d\vec{x}}{dt} \times \vec{B}\right)$$



$q = -e$ for electron w/ $e > 0$

the eom follows from Hamiltonian

$$H = \frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + q\phi$$

q – charge of charged particle

use Hamilton's eq^{n}'s

$\vec{A}(\vec{x},t) = \vec{A}$

$\phi(\vec{x},t) = \phi$

$$\vec{p}\cdot\vec{p} - \frac{q}{c}\underbrace{\left(\vec{p}\cdot\vec{A} + \vec{A}\,\vec{p}\right)}_{\text{take to be right form}} + \left(\frac{q}{c}\right)^2 \vec{A}\cdot\vec{A}$$

in classical mechanics, cool

in QM: in general $[\vec{p}, \vec{A}(\vec{x},t)] \neq 0$

$$[p_i, A_j]\, f(\vec{x})$$

$$= -i\hbar \frac{\partial}{\partial x_i}\left(A_j(\vec{x},t) f(\vec{x})\right) - A_j(\vec{x},t)\left(-i\hbar \frac{\partial}{\partial x_i} f(\vec{x})\right)$$

$$= -i\hbar \frac{\partial A_j}{\partial x_i} f(\vec{x}) \underbrace{- i\hbar A_j \frac{\partial f(\vec{x})}{\partial x_i} + i\hbar A_j \frac{\partial f(\vec{x})}{\partial x_i}}_{\text{cancel!}}$$

$$= -i\hbar \frac{\partial A_j}{\partial x_i} f(\vec{x})$$

① Check H leads to eom

② What happens when we use H in QM?

$$[p_i, A_j] = -i\hbar \frac{\partial A_j}{\partial x_i}$$

if $\nabla A = 0$ → woo, this is 0!

Consider SE

$$i\hbar \frac{\partial \psi(\vec{x},t)}{\partial t} = H\,\psi(\vec{x},t) = \left(\frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}(\vec{x},t)\right)^2 + q\,\phi(\vec{x},t)\right)\psi(\vec{x},t)$$

$\vec{p}$: $-i\hbar\nabla$ in pos^{n} space

$\vec{x}$ in pos^{n} space

This eq^{n} is not invariant under gauge transformations of $\vec{A}$, ϕ

same $\vec{E}$ & $\vec{B}$, different SE in QM: $\quad \psi \to \psi' = e^{iq\Lambda(\vec{x},t)/\hbar c}\,\psi$

$(\vec{A}', \phi', \psi')$ has same physics as $(\vec{A}, \phi, \psi)$

Define $\quad D_0 = \dfrac{\partial}{\partial t} + \dfrac{iq\phi}{\hbar}$

$\vec{D} = \nabla - \dfrac{iq}{\hbar c}\vec{A}$ $\Big\}$ covariant derivatives

SE. $\quad i\hbar D_0 \psi = -\dfrac{\hbar^2}{2m}\vec{D}\cdot\vec{D}\,\psi$

$$D_0'\psi' = \left(\frac{\partial}{\partial t} + \frac{iq}{\hbar}\phi'\right)\left(e^{iq\Lambda/\hbar c}\psi\right)$$

$$= \frac{\partial}{\partial t}\left(e^{iq\Lambda/\hbar c}\psi\right) + \frac{iq}{\hbar}e^{iq\Lambda/\hbar c}\psi$$

$$= \frac{iq}{\hbar c}\frac{\partial\Lambda}{\partial t}e^{iq\Lambda/\hbar c}\psi + e^{iq\Lambda/\hbar c}\frac{\partial\psi}{\partial t} + \frac{iq}{\hbar}e^{iq\Lambda/\hbar c}\psi$$

$$= \left(\frac{iq}{\hbar c}\frac{\partial\Lambda}{\partial t} + \frac{\partial}{\partial t} + \frac{iq}{\hbar}\right)e^{iq\Lambda/\hbar c}\psi$$