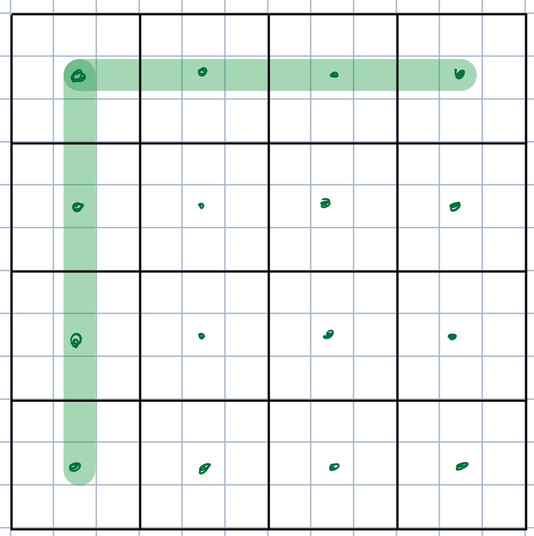


$k \times k$

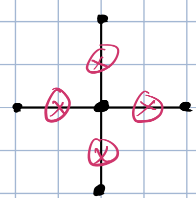
v • lattice points/vertices

p • dual lattice points

spin ½ - 2 state system to each •—•

$A_v = \prod_{j \in star(v)} X_j$

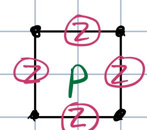
$Z \leftrightarrow X$ compared to last time



$\sigma_x |\pm\rangle = |\mp\rangle$

$\sigma_z |\pm\rangle = \pm|\pm\rangle$

to each p $\quad B_p = \prod_{j \in bndy(p)} Z_j$ $\quad Z \leftrightarrow X$ compared to before



As before, $A_v^2 = 1 = B_p^2$ $\quad \prod_v A_v = \mathbb{1}$ $\quad \prod_p B_p = \mathbb{1}$ $\quad [A_v, B_p] = 0$ any v & p

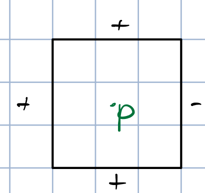
same counting, so space of $|\psi\rangle$ w/ $A_v|\psi\rangle = B_p|\psi\rangle = |\psi\rangle$ all v,p is 4-D

Work in terms of $\sigma_z(Z)$ eigenstates $\quad Z|s\rangle = s|s\rangle$ $\quad s = \pm 1$ for each spin ½

$S = \{s_1, \ldots, s_{2k^2}\}$ that label all states if they are all Z eigenstates

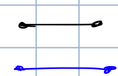
each •—• has $\pm$ associated w/ it

For each p, define $\quad \omega_p \equiv \prod_{j \in bndy(p)} s_j$



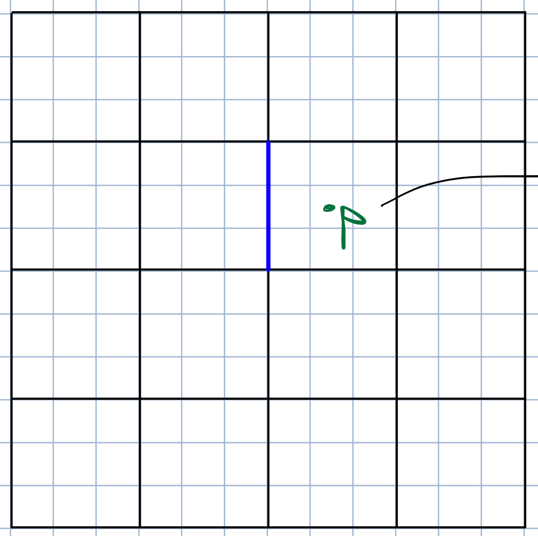
$\omega_p = (+1)^3(-1) = -1$

demanding $B_p|\psi\rangle = |\psi\rangle \longrightarrow \omega_p = +1$ for any p $\longrightarrow$ even # of -1 eigenstates surrounding each p

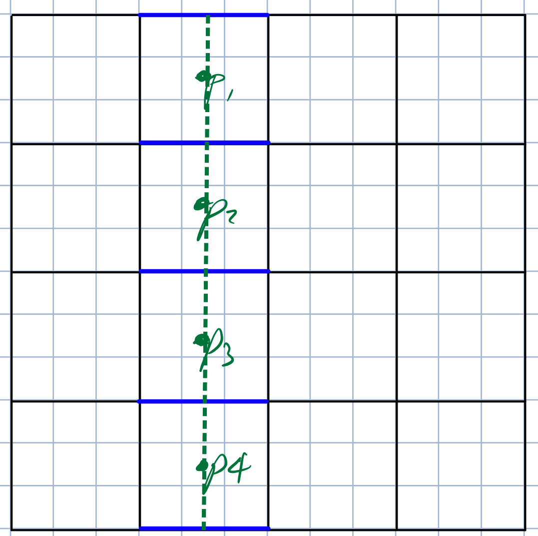


$+1$

-1



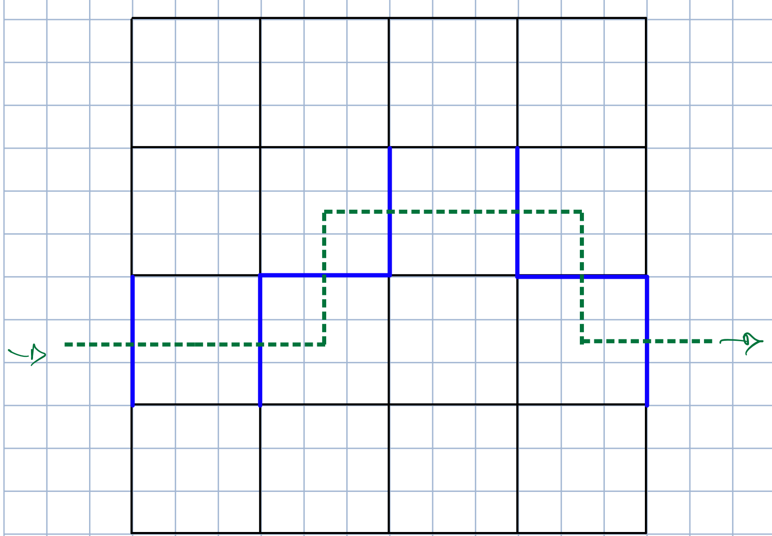
$\omega_p = -1$



$\omega_{p_1} = \omega_{p_2} = \omega_{p_3} = +1$

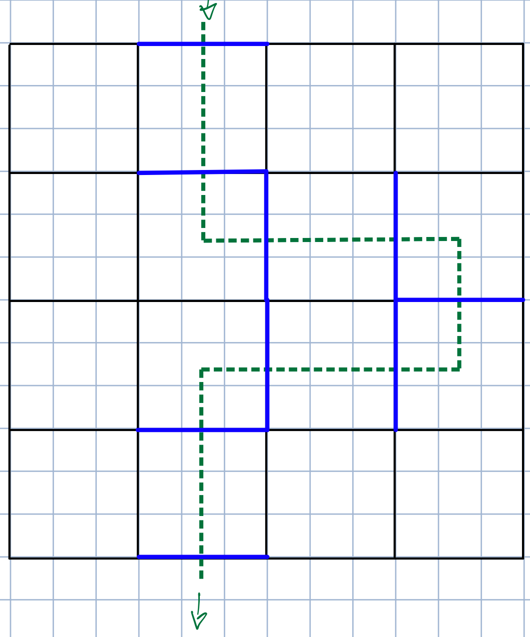
$B_p|\psi\rangle = |\psi\rangle$ for all p

closed loop connecting dual lattice points



$B_p|\psi\rangle = |\psi\rangle$

closed loop!

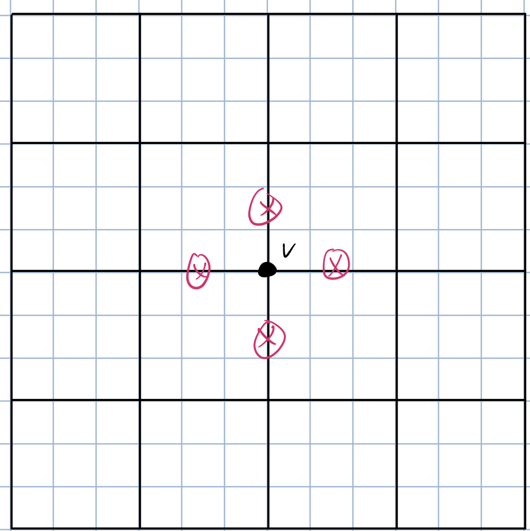


draw

make edge crossed

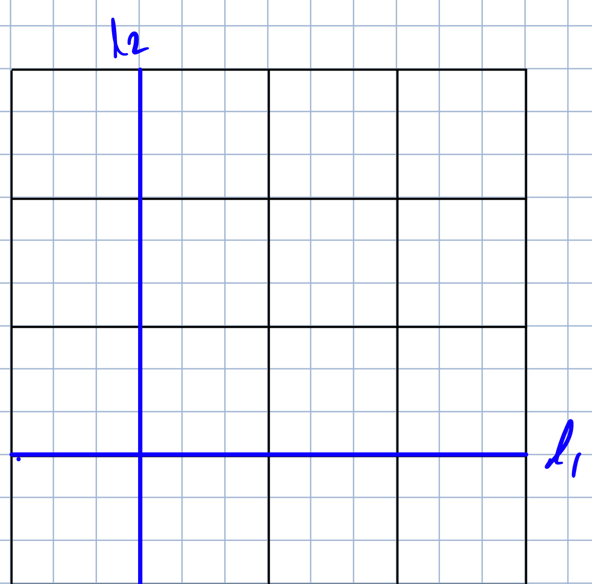
even # of purple lines around each p
& closed loops on dual lattices

What does A_v do to one of these configurations?

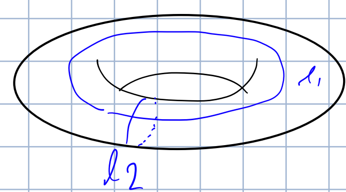


flips $|\pm\rangle \rightarrow |\mp\rangle$ on an even # of lines

acting w/ A_v reroutes the loop or adds topologically trivial loops



2 indpt. closed loop l_1 & l_2



$$W_1 \equiv \prod_{j \in l_1} S_j \quad \in \{-1, +1\}$$

$$W_2 \equiv \prod_{j \in l_2} S_j \quad \in \{-1, +1\}$$

given configuration satisfying $B_p|\psi\rangle = |\psi\rangle$, acting w/ A_v doesn't change W_1 or W_2

4 classes of ground states $(w_1, w_2) = (1,1), (1,-1), (-1,1), (-1,-1)$

$$|\psi\rangle_{1,1} = \sum_{\substack{s,\, \omega_p(s)=1 \text{ all } p \\ \omega_1 = \omega_2 = 1}} |s\rangle \quad |s\rangle \to |s_1\rangle \otimes |s_2\rangle \ldots$$

$$|\psi\rangle_{1,-1} = \sum_{\substack{s,\, \omega_p(s)=1 \text{ all } p \\ \omega_1 = 1,\, \omega_2 = -1}} |s\rangle$$

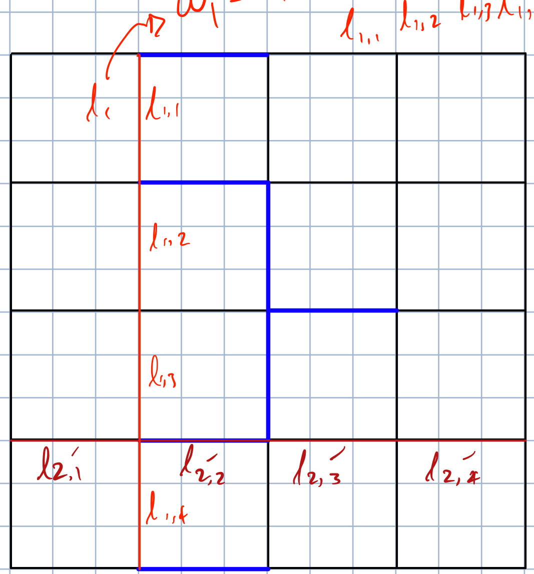
$|{+}{+}\rangle, |{+}{-}\rangle, |{-}{+}\rangle, |{-}{-}\rangle$

Define Hamiltonian $H = -\sum_v A_v - \sum_p B_p$

"ground state" has all $A_v = 1$ & $B_p = 1$ $\to$ $A_v|\psi\rangle = |\psi\rangle$ & $B_p|\psi\rangle = |\psi\rangle$

but there are 4 of them

$\ell_1 \to \omega_1 = +1 = \underset{\ell_{1,1}}{(+1)}\underset{\ell_{1,2}}{(+1)}\underset{\ell_{1,3}}{(+1)}\underset{\ell_{1,4}}{(+1)}$



$\ell_2 \to \omega_2 = -1 = \underset{\ell_{2,1}}{(+1)}\underset{\ell_{2,2}}{(-1)}\underset{\ell_{2,3}}{(+1)}\underset{\ell_{2,4}}{(-1)}$

acting with A_v will change even # of the —○ or —○
so it will stay same