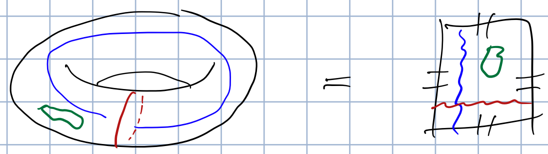
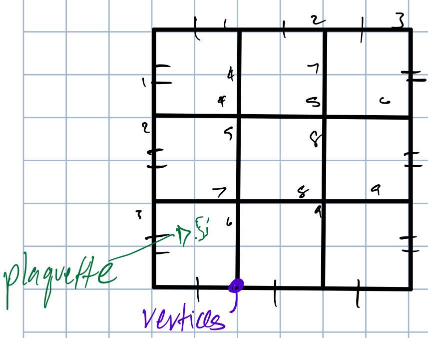
Midterm → take home 2 hour window

Toric Code





$k \times k$ grid of squares
(3x3)

spin ½ particle 2 state system $\mathbb{C}^2$

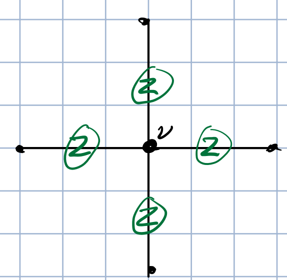
$9+9 = 18 = 2\cdot 3^2$, $2\times k^2$ for $k=3$ $(\mathbb{C}^2)^{\otimes 18}$

$\circ\!\!-\!\!\textcircled{X}\!\!-\!\!\circ = \sigma_x |\psi\rangle$ on this vertex

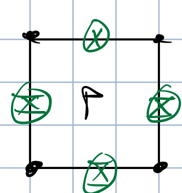
$\circ\!\!-\!\!-\!\!\circ = |0\rangle$ "unfilled" $Z|0\rangle = |0\rangle$
$\circ\!\!-\!\!-\!\!\circ = |1\rangle$ "filled" $Z|1\rangle = -|1\rangle$

For each vertex v, can associate operator

$$A_v = \prod_{j\in \text{star}(v)} Z_j = \text{product of 4 } Z\text{'s}$$



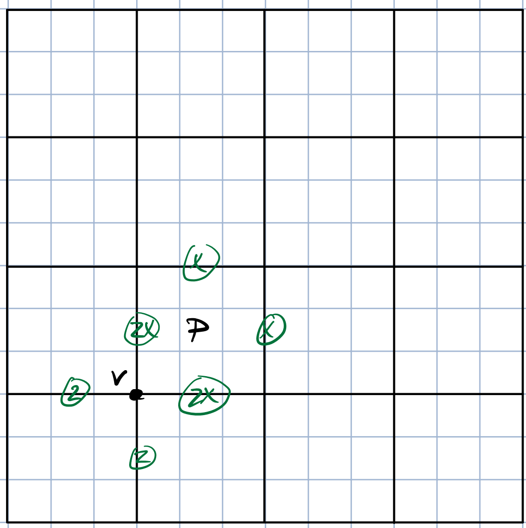
$$B_p = \prod_{j\in \text{bndy}(p)} X_j = \text{product of 4 } X\text{'s}$$



P plaquette

① $A_v^2 = 1$ all v
② $B_p^2 = 1$ all p → A_v, B_v have eigenvalues ± 1

③ $[A_v, B_p] = 0$ for all v, p



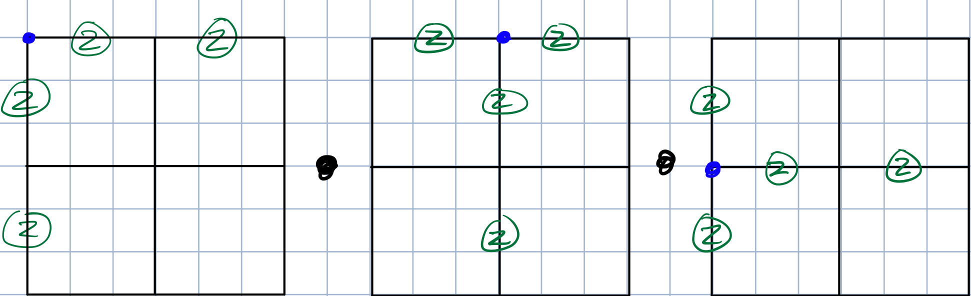
$[\sigma_x \otimes \sigma_x,\ \sigma_z \otimes \sigma_z] = 0$

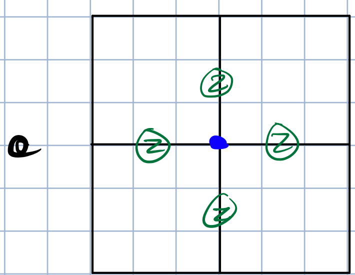
Stabilizer code space $V_S = \{\ |\psi\rangle \in (\mathbb{C}^2)^{2k^2} \mid B_p|\psi\rangle = |\psi\rangle, A_v|\psi\rangle = |\psi\rangle \text{ all } v, p\}$

1. how big is V_S?
2. what do errors "look like"?
3. what do elements in V_S "look like"?

$\prod_v A_v = \mathbb{1}$
$\prod_p B_p = \mathbb{1}$

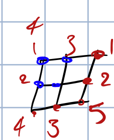
$\prod_v A_v =$





$= \mathbb{1}$

only



b/c $1=1$, $2=2$, $3=3$
$4=4=1$
$5=1$

each acted on w/ σ_z^2

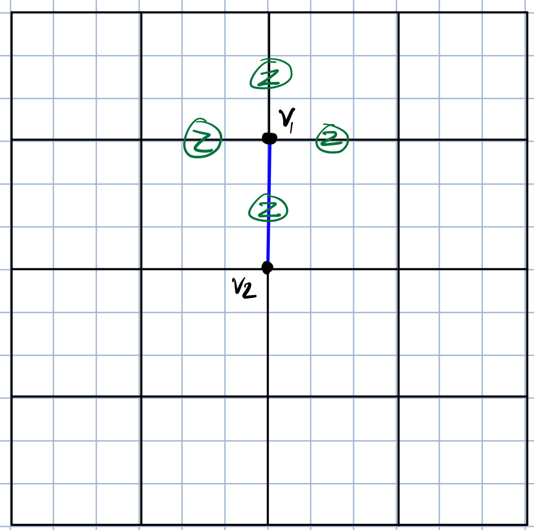
$\longrightarrow$ k^2 A_v, k^2 B_p, 2 constraints $\prod_p B_p = \prod_v A_v = \mathbb{1}$

$\therefore$ V_S has dimensions $2^{2k^2-(2k^2-2)} = 2^2 = 4$ 2 qubits!

What do states w/ $A_v|\psi\rangle = |\psi\rangle$ look like?

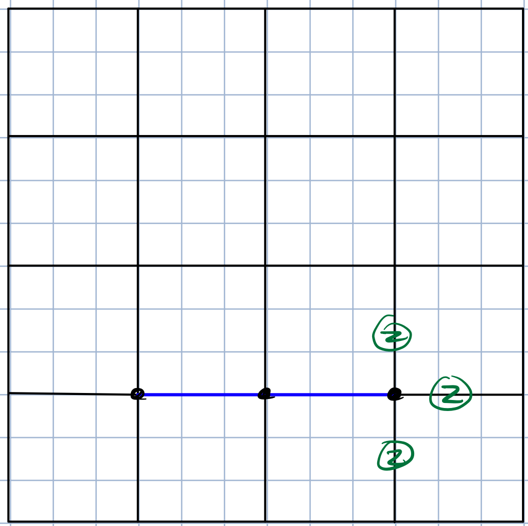
$$|0\rangle \otimes |0\rangle \otimes \ldots \otimes |0\rangle = (|0\rangle)^{\otimes 2L^2} = |\text{unfilled}\rangle$$

$$A\,|\text{unfilled}\rangle = |\text{unfilled}\rangle$$

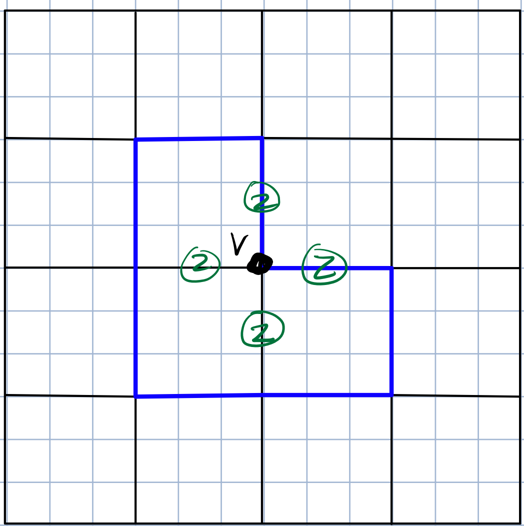


$$A_{v_1}|\psi\rangle = -|\psi\rangle$$

$$A_{v_2}|\psi\rangle = -|\psi\rangle$$

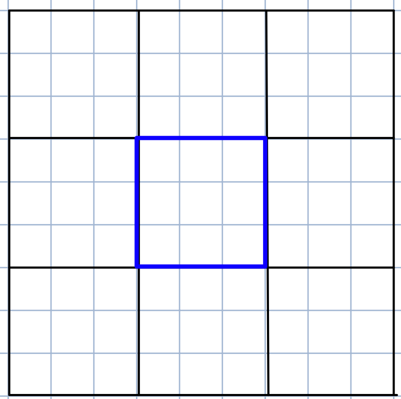


make closed loop!

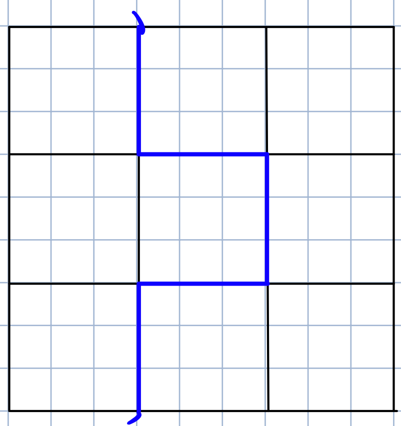


$$A_v = \pm 1$$

$$A_v \,|\text{closed loop of filled state}\rangle = (+1)\,|\text{closed loop of filled state}\rangle$$

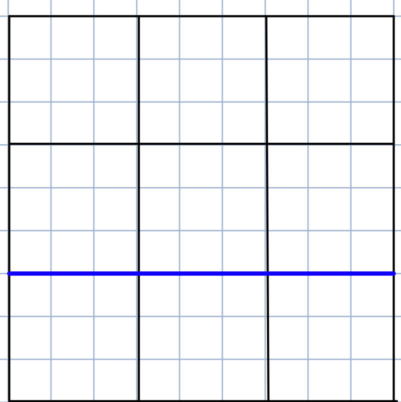


contractable



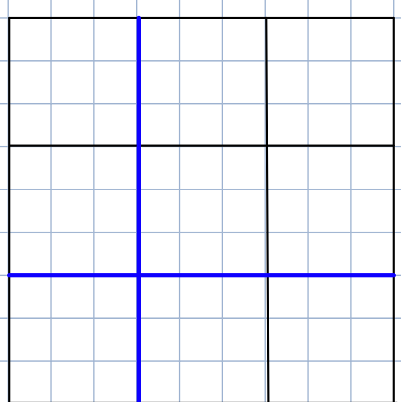
non contractable

vertical wind



non contractable

horizontal wind



combination of other 2 winds

Topology

What does B_p do to closed loop?

$$B_p = \prod_j X_j \quad j \in \mathrm{bndy}(p)$$

flips ~~states~~!

B_p's on closed loop is new closed loop in same topological class

$B_p |\text{closed loop}\rangle = |\text{new closed loop}\rangle$