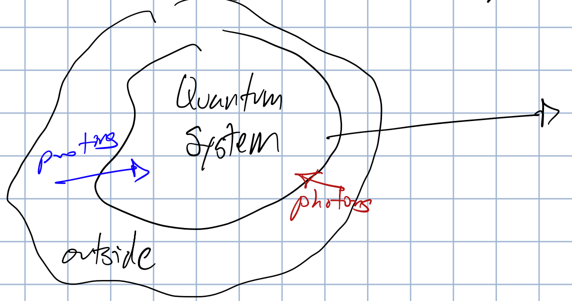
Civil disobedience: Zoom office hours? :)

Upload psets thru Canvas

Why do we need density matrices?

not really taught, but need for IRL experiments

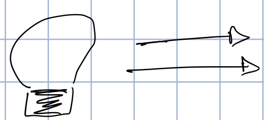


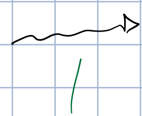
can isolate system, but will still be entanglement w/ outside world

$\longrightarrow$ uncertainty

Alice Bob

Unpolarized light





$\frac{1}{2}\,|L\rangle$

$\frac{1}{2}\,|R\rangle$

can approximate w/ mixed state

Density operator w.r.t. bases $|\psi\rangle$ has form

$$\longrightarrow \rho = \sum p_i\, |\psi_i\rangle\langle\psi_i| \qquad \sum_i p_i = 1$$

$$\langle\psi_k|\rho|\psi_l\rangle = \rho_k\, \delta_{kl} \longrightarrow \begin{pmatrix} p_1 & & & 0 \\ & p_2 & & \\ & & \ddots & \\ 0 & & & p_N \end{pmatrix}$$

Example: $|+\rangle, |-\rangle$

$$\rho = \tfrac{1}{2}|+\rangle\langle+| + \tfrac{1}{2}|-\rangle\langle-|$$

consider states: $|\alpha\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$ & $|\beta\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)$

$\langle\alpha|\alpha\rangle = 1$
$\langle\alpha|\beta\rangle = 0$
$\langle\beta|\beta\rangle = 1$

$$\rho' = \tfrac{1}{2}|\alpha\rangle\langle\alpha| + \tfrac{1}{2}|\beta\rangle\langle\beta|$$

$$= \tfrac{1}{2}\left(\tfrac{1}{\sqrt{2}}(|+\rangle + |-\rangle)\tfrac{1}{\sqrt{2}}(\langle+| + \langle-|)\right) + \tfrac{1}{2}\left(\tfrac{1}{\sqrt{2}}(|+\rangle - |-\rangle)\tfrac{1}{\sqrt{2}}(\langle+| - \langle-|)\right)$$

$$= \tfrac{1}{2}|+\rangle\langle+| + \tfrac{1}{2}|-\rangle\langle-| = \rho$$

ρ doesn't determine unique sets of probabilities

unitary matrix

Check $\rho = \{p_i, |\psi_i\rangle\}$ $\quad \rho' = \{q_i, |\phi_i\rangle\}$

$\rho = \rho'$ iff $\quad \sqrt{p_i}\,|\psi_i\rangle = \sum_j U_{ij}\sqrt{q_j}\,|\phi_j\rangle$

i is fixed index, not summed over

Two general properties

① Trace of $\rho = 1$

$$\mathrm{Tr}(\rho) = 1$$

$$\mathrm{Tr}(\rho) = \sum_j \langle\psi_j|\rho|\psi_j\rangle$$

$$= \sum_j \langle\psi_j|\left(\sum_i p_i |\psi_i\rangle\langle\psi_i|\right)|\psi_j\rangle$$

$$= \sum_i p_i\,\delta_{ij}\delta_{ij} = \sum_i p_i = 1$$

we're assuming it is finite

② ρ is a positive element

$\langle\varphi|\rho|\varphi\rangle \geq 0$ for any $|\varphi\rangle \in \mathcal{H}$

$$\to \sum_i p_i \langle\varphi|\psi_i\rangle\langle\psi_i|\varphi\rangle = \sum_i p_i\, \underbrace{|\langle\varphi|\psi_i\rangle|^2}_{\geq 0}$$

$0 \leq p_i$

→ Any Trace 1 positive operator ρ can be written as: $\rho = \sum_i \lambda_i |\chi_i\rangle\langle\chi_i|$
s.t. $\sum_i \lambda_i = 1$ & $\lambda_i \in \mathbb{R}$

Entanglement & density operators!

$V_A \otimes V_B$ $\qquad$ $V_A \in \mathbb{C}^2$
$\qquad\qquad\qquad\quad$ $V_B \in \mathbb{C}^2$

↳ basis

$|+\rangle_A \otimes |+\rangle_B \to |++\rangle$
$|+\rangle_A \otimes |-\rangle_B \to |+-\rangle$
$|-\rangle_A \otimes |+\rangle_B \to |-+\rangle$
$|-\rangle_A \otimes |-\rangle_B \to |--\rangle$

State in $\mathbb{C}^2 \otimes \mathbb{C}^2$ was entangled if it could <u>not</u> be written in the form $|\chi\rangle_A \otimes |\chi\rangle_B$

new defn!

consider states: $|\psi\rangle = |+\rangle_A \otimes |+\rangle_B = |++\rangle$ $\quad$ & $|\chi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_A \otimes |-\rangle_B - |-\rangle_A \otimes |+\rangle_B \right)$

what is probability A measures +1 for $\sigma_3 \to \sigma_3 = (\sigma_3)_A \otimes \mathbb{1}_B$

2 states: $|++\rangle$, $|+-\rangle$

for $|\psi\rangle$ prob: $P(A, +1) = |\langle ++|\psi\rangle|^2 = 1$

for $|\chi\rangle$ $\quad P(A, +1) = |\langle ++|\chi\rangle|^2 + |\langle +-|\chi\rangle|^2 = 0 + \frac{1}{2} = \frac{1}{2}$

for any $V_A \otimes V_B$ can construct density operator acting on V_A by taking trace of V_B on any state $|\phi\rangle$

$\to \rho_A = \mathrm{Tr}_{V_B}\left(|\phi\rangle\langle\phi| \right)$

↳ any state $|\phi\rangle \in V_A \otimes V_B$

$|i\rangle_A$ basis for V_A
$|j\rangle_B$ basis for V_B

$\mathrm{Tr}_{V_B}(\mathcal{O}) = \sum_j {}_B\langle j|\mathcal{O}|j\rangle_B$

$\mathrm{Tr}_{V_A}(\mathcal{O}) = \sum_i {}_A\langle i|\mathcal{O}|i\rangle_A$

example: $\sigma = |\psi\rangle\langle\psi|$ for unentangled state

$$\rho_A = Tr_{V_B}(\sigma) = {}_B\langle +|\sigma|+\rangle_B + {}_B\langle -|\sigma|-\rangle_B$$

$$= \underbrace{{}_B\langle +|}\,(|+\rangle_A \otimes \overbrace{|+\rangle_B)}^{1}(\,{}_A\langle +| \otimes \overbrace{{}_B\langle +|)\,|+\rangle_B}^{1}$$

$$+ \underbrace{{}_B\langle -|}\,(|+\rangle_A \otimes \overbrace{|+\rangle_B)}^{0}(\,{}_A\langle +| \otimes \overbrace{{}_B\langle +|)\,|-\rangle_B}^{0}$$

$$= |+\rangle_A\,{}_A\langle +| + 0$$

example 2: $\sigma = |\chi\rangle\langle\chi|$

$$\rho_A = Tr_{V_B}(|\chi\rangle\langle\chi|) = \sum_i {}_B\langle i|\chi\rangle\langle\chi|i\rangle_B$$

$$= {}_B\langle +|\chi\rangle\langle\chi|+\rangle_B + {}_B\langle -|\chi\rangle\langle\chi|-\rangle_B$$

$$= {}_B\langle +|\tfrac{1}{\sqrt2}(|+\rangle_A\otimes|-\rangle_B - |-\rangle_A\otimes|+\rangle_B)\ \tfrac{1}{\sqrt2}(\langle +|_A\otimes\langle -|_B - \langle -|_A\otimes\langle +|_B)\,|+\rangle_B$$

$$+ {}_B\langle -|\tfrac{1}{\sqrt2}(|+\rangle_A\otimes|-\rangle_B - |-\rangle_A\otimes|+\rangle_B)\ \tfrac{1}{\sqrt2}(\langle +|_A\otimes\langle -|_B - \langle -|_A\otimes\langle +|_B)\,|-\rangle_B$$

$$= \{\tfrac{1}{\sqrt2}|+\rangle_A \otimes \overset{0}{\cancel{{}_B\langle +|-\rangle_B}} - \tfrac{1}{\sqrt2}|-\rangle_A \otimes {}_B\langle +|+\rangle_B\} \cdot \{\tfrac{1}{\sqrt2}\,{}_A\langle +| \otimes \overset{0}{\cancel{{}_B\langle -|+\rangle_B}} - \tfrac{1}{\sqrt2}\,{}_A\langle -| \otimes {}_B\langle +|+\rangle_B\}$$

$$+ \{\tfrac{1}{\sqrt2}|+\rangle_A \otimes {}_B\langle -|-\rangle_B - \tfrac{1}{\sqrt2}|-\rangle_A \otimes \overset{0}{\cancel{{}_B\langle -|+\rangle_B}}\} \cdot \{\tfrac{1}{\sqrt2}\,{}_A\langle +| \otimes {}_B\langle -|-\rangle_B - \tfrac{1}{\sqrt2}\,{}_A\langle +| \otimes \overset{0}{\cancel{{}_B\langle +|-\rangle_B}}\}$$

$$= -\tfrac{1}{\sqrt2}|-\rangle_A \cdot -\tfrac{1}{\sqrt2}\langle -|_A + \tfrac{1}{\sqrt2}|+\rangle_A\ \tfrac{1}{\sqrt2}\,{}_A\langle +| = \tfrac12 |-\rangle_A\,{}_A\langle -| + \tfrac12 |+\rangle_A\,{}_A\langle +|$$

$$\longrightarrow \rho_A = \begin{pmatrix} \tfrac12 & 0 \\ 0 & \tfrac12 \end{pmatrix} = \tfrac12 |+\rangle_A\,{}_A\langle +| + \tfrac12 |-\rangle_A\,{}_A\langle -|$$

$|\psi\rangle \in V_A \otimes V_B$ is entangled if $\rho_A = Tr_{V_B}(|\psi\rangle\langle\psi|)$ has more than 1 nonzero eigenvalues