More QEC

phase flip

general 1-qubit QEC w/ Shor code embed $\mathbb{C}^2 \to (\mathbb{C}^2)^{\otimes 9}$

CSS codes - connect QEC w/classical codes

toric codes

$\to$ QM + EM

Midterm (next Friday)

stuff up to

Phase Flip

$|0\rangle \to |0\rangle$

$|1\rangle \to -|1\rangle$

$a|0\rangle + b|1\rangle \to a|0\rangle - b|1\rangle$ quantum error

$|0\rangle \equiv |+,z\rangle$ & $|1\rangle \equiv |-,z\rangle$

$|+\rangle \equiv |+,x\rangle$ & $|-\rangle \equiv |-,x\rangle$

$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$

$|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

phase flip error $|0\rangle \to |0\rangle$ $|1\rangle \to -|1\rangle$

takes $|+\rangle \to |-\rangle$ & $|-\rangle \to |+\rangle$ it's bit flip error in $|\pm\rangle$ basis

change of basis from $|0\rangle$ & $|1\rangle$ to $|+\rangle$ & $|-\rangle$ accomplished by H Hadamard Gate

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad H\begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} |0\rangle + |1\rangle \\ |0\rangle - |1\rangle \end{pmatrix} = \begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix}$$

Exercise: by acting w/H in right places show how to use bit flip error correction to connect phase flip errors

Using an encoding of one qubit into 3 qubit space we can detect & correct one bit flip <u>or</u> one phase flip error

σ_y: $|0\rangle \to -i|1\rangle$ $|1\rangle \to i|0\rangle$

for bit flip correction, we measured P_0, P_1, P_2, P_3 (last class)

Shorter procedure. measure: $Z_1 Z_2 = \sigma_z \otimes \sigma_z \otimes \mathbb{1}$

$Z_2 Z_3 = \mathbb{1} \otimes \sigma_z \otimes \sigma_z$

can check $Z_1 Z_2 = \left(|00\rangle\langle 00| + |11\rangle\langle 11|\right) \otimes \mathbb{1} - \left(|10\rangle\langle 10| + |01\rangle\langle 01|\right) \otimes \mathbb{1}$

$\hookrightarrow$ tells you if 1^{st} & 2^{nd} qubits are same $(+1)$ or different (-1)

	$Z_1 Z_2$	$Z_2 Z_3$
no error	1	1
1^{st} bitflip	-1	1
2^{nd} bit flip	-1	-1
3^{rd} bit flip	1	-1

$\to$ act w/ $X_2 = \mathbb{1} \otimes \sigma_x \otimes \mathbb{1}$ to correct error

Shor code – a 9 qubit encoding of one qubit

encode $a|0\rangle + b|1\rangle \to a|\bar{0}\rangle + b|\bar{1}\rangle$

$$|\bar{0}\rangle = \frac{1}{2\sqrt{2}}\left(|000\ 000\ 000\rangle + |000\ 000\ 111\rangle + 6 \text{ more} \right)$$

$$= \frac{1}{2\sqrt{2}}\left(\underbrace{(|000\rangle + |111\rangle)}_{\text{block 1}} \otimes \underbrace{(|000\rangle + |111\rangle)}_{\text{block 2}} \otimes \underbrace{(|000\rangle + |111\rangle)}_{\text{block 3}} \right)$$

$$= \frac{1}{2\sqrt{2}}\left(|000\rangle + |111\rangle \right)^{\otimes 3}$$

$$|\bar{1}\rangle = \frac{1}{2\sqrt{2}}\left(|000\rangle - |111\rangle \right)^{\otimes 3}$$

encode $|0\rangle$, encoding as $\sim |{+}{+}{+}\rangle$ as in phase flip error

each $\frac{|000\rangle + |111\rangle}{\sqrt{2}}$ as in bit flip error

3 types of errors

1. bit flip — act X_i — $i = 1, 2, \ldots, 9$
2. phase flip — act Z_i — $i = 1, 2, \ldots, 9$
3. bit & phase flip — act Y_i — $i = 1, 2, \ldots, 9$

Bit flip in 1^{st} qubit

$$|\psi_E\rangle = \frac{a}{2\sqrt{2}}\Big((|100\rangle + |011\rangle) \otimes (|000\rangle + |111\rangle)^{\otimes 2}\Big)$$
$$+ \frac{b}{2\sqrt{2}}\Big((|100\rangle - |011\rangle) \otimes (|000\rangle - |111\rangle)^{\otimes 2}\Big)$$

Measure $Z_1 Z_2 \longrightarrow -1$ (1^{st} & 2^{nd} are diff)

$Z_2 Z_3 \longrightarrow 1$ (2^{nd} & 3^{rd} are same)

$\Rightarrow$ implies first bit flip

$$X_2 |\psi_E\rangle = |\psi\rangle = a|\bar{0}\rangle + b|\bar{1}\rangle$$

in general, measure $Z_1 Z_2$ & $Z_2 Z_3$ (1st block), $Z_4 Z_5$ & $Z_5 Z_6$ (2nd block), $Z_7 Z_8$ & $Z_8 Z_9$ (3rd block)

Phase flip in 1^{st} qubit

$|0\rangle \to |0\rangle$ $|1\rangle \to -|1\rangle$

$$|\psi_E\rangle = \frac{a}{2\sqrt{2}}\Big((|000\rangle - |111\rangle) \otimes (|000\rangle + |111\rangle)^{\otimes 2}\Big)$$

$-|1\rangle \to -\cdot -|1\rangle \to +|1\rangle$

$$+ \frac{b}{2\sqrt{2}}\Big((|000\rangle + |111\rangle) \otimes (|000\rangle + |111\rangle)^{\otimes 2}\Big)$$

Same expression for any in block. instead of locating qubit, we locate block & fix block

Measure $X_1 X_2 X_3 X_4 X_5 X_6 = (X_1 X_2 X_3)(X_4 X_5 X_6)(\mathbb{1}\mathbb{1}\mathbb{1})$

$\vdots$

$X_4 X_5 X_6 X_7 X_8 X_9 = (\mathbb{1}\mathbb{1}\mathbb{1})(X_4 X_5 X_6)(X_7 X_8 X_9)$