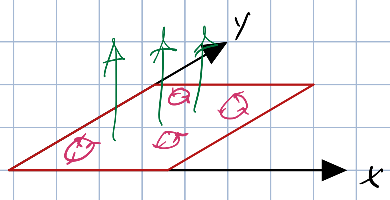


$\vec{B} = B_0 \hat{e}_z$

classical equations of motion: $\vec{F} = m\vec{a}$

$$m\frac{d\vec{v}}{dt} = q\,\frac{\vec{v}}{c} \times \vec{B}$$

electron $q = -e$ $e > 0$

$$m\frac{d^2x}{dt^2} = -\frac{eB}{c}\frac{dy}{dt}$$

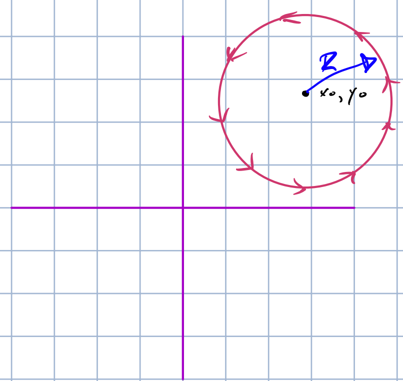
$$m\frac{d^2y}{dt^2} = \frac{eB}{c}\frac{dx}{dt}$$

$$x(t) = x_0 - R\sin(\omega_0 t + \phi)$$

$$y(t) = y_0 + R\cos(\omega_0 t + \phi)$$

where ϕ, R, y_0, x_0 are arbitrary, real params determined by $x, y, \frac{dx}{dt}, \frac{dy}{dt}$ @ some time

cyclotron frequency $\rightarrow$ $\omega = -\frac{eB}{mc}$



counter clockwise motion

motion is not invariant under time reversal

in Maxwell's eq'^ns, $\vec{E}$ & $\vec{B}$ determined by ρ & $\vec{j}$

when reversing time, $\rho \rightarrow \rho$ & $\vec{j} \rightarrow -\vec{j}$

results in $\vec{E} \rightarrow \vec{E}$ & $\vec{B} \rightarrow -\vec{B}$

What happens in QM?

motion in 2 directions: (x, y) & (p_x, p_y)

$$H = \frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)\left(\vec{p} - \frac{q}{c}\vec{A}\right) = \frac{1}{2m}\left(\Pi_x^2 + \Pi_y^2\right)$$

$$B = (\vec{B})_z = \frac{\partial}{\partial x}(A_y) - \frac{\partial}{\partial y}(A_x) = \text{constant}$$

$\Pi_x = p_x - \frac{q}{c}A_x$ $[p_x, p_y] = 0$

$\Pi_y = p_y - \frac{q}{c}A_y$ $[A_x, A_y] = 0$

$$[\Pi_x, \Pi_y] = \left[-i\hbar\frac{\partial}{\partial x} - \frac{q}{c}A_x,\ -i\hbar\frac{\partial}{\partial y} - \frac{q}{c}A_y\right]$$

$$= i\hbar\frac{q}{c}\left(\frac{\partial}{\partial y}(A_x) - \frac{\partial}{\partial x}(A_y)\right)$$

$$= i\hbar\frac{q}{c}\cdot B$$

constant!

$[A_x, A_y] = 0$

$\left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right] = 0$

$$\tilde{Q} = \frac{c}{qB}\cdot\Pi_x \qquad \tilde{P} = \Pi_y$$

$\omega_B = \frac{qB}{mc}$ $[\tilde{Q}, \tilde{P}] = i\hbar$

$$H = \frac{1}{2m}\tilde{P}^2 + \frac{1}{2}m\omega_B^2\tilde{Q}^2$$

SHO! :)

already know energies for SHO: $E_n = (n+\frac{1}{2})\hbar\omega_B$

each energy level is infinitely degenerate for an infinite system

can look @ coord. space wave $f^{\underline{n}}$s

need to choose $\hat{x}$ & $\hat{y}$ → choose a gauge

① Landau gauge (now) $A_x = 0$ $A_y = Bx$
② pset :)

$$H = \frac{1}{2m}\left(p_x^2 + (p_y - \tfrac{qBx}{c})^2\right) = \frac{1}{2m}\left(p_x^2 + p_y^2 - \frac{2qBxp_y}{c} + \frac{q^2B^2x^2}{c^2}\right)$$

H has no y dependence! $[H, p_y] = 0$ → can find simultaneous eigenstates of H & p_y

we can look for H eigenstates of form:

$$\psi_E(x,y) = e^{iky} v_E(x)$$

$$p_y \psi_E = -i\hbar \frac{d}{dy}\left(e^{iky} v_E(x)\right) = \hbar k e^{iky} v_E(x) = \hbar k \psi_E$$

$\hbar k$ – momentum eigenvalue

k can be whatever value → infinite sol$^{\underline{n}}$s

substitute into time indpt. S.E.

$$H\psi_E = E\psi_E$$

get equation for $v_E(x)$

$$\left[\frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega_B^2}{2}\left(x - x_0(k)\right)^2\right] v_E(x) = E\, v_E(x)$$

$$x_0(k) = \frac{\hbar c k}{qB}$$

Hamiltonian for 1-D SHO located @ $x_0(k)$

$$v_E(x) = U_n(x - x_0(k))$$

U_n – nth SHO energy eigenstate = (Hermite polynomials) exp( )

$$\psi_n^{(k)}(x,y) = e^{iky} U_n(x - x_0(k))$$

n – energy $0, 1, 2, \ldots$
k – p_y eigenvalue $k_y \in \mathbb{R}$

SHO grnd state $e^{x^2(\,)}$

1st excited state $cx \cdot e^{x^2(\,)}$

2nd exited state $P(x)\, e^{x^2(\,)}$