November! woo!

last time:

① $H = \frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + q\phi$ Hamiltonian for particle of charge q in $\vec{B} = \nabla \times \vec{A}$ & $\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$

② SE is invariant under

$$\vec{A} \to \vec{A}' = \vec{A} + \vec{\nabla}\Lambda$$
$$\phi \to \phi' = \phi - \frac{1}{c}\frac{\partial \Lambda}{\partial t}$$
$$\psi \to \psi' = e^{\frac{iq\Lambda}{\hbar c}}\psi$$

$\Lambda(\vec{x}, t)$

can write 2 covariant Derivatives

$$D_0 = \frac{\partial}{\partial t} + \frac{iq}{\hbar}\phi$$
$$\vec{D} = \vec{\nabla} - \frac{iq}{\hbar c}\vec{A}$$

SE: $i\hbar \frac{\partial}{\partial t}\psi = H\psi \to i\hbar D_0 \psi(\vec{x}, t) = -\frac{\hbar^2}{2m}\vec{D}\cdot\vec{D}\psi$

$$D_0\psi \to D_0'\psi' = e^{iq\Lambda/\hbar c} D_0\psi$$

$$\vec{D}\psi \to \vec{D}'\psi' = \left(\vec{\nabla} - \frac{iq}{\hbar c}\vec{A}'\right)e^{iq\Lambda/\hbar c}\psi$$
$$= \left(\vec{\nabla} - \frac{iq}{\hbar c}\vec{A} - \frac{iq}{\hbar c}\vec{\nabla}\Lambda\right)e^{iq\Lambda/\hbar c}\psi$$
$$= \vec{\nabla}\left(e^{iq\Lambda/\hbar c}\psi\right) - \frac{iq}{\hbar c}\vec{A}e^{iq\Lambda/\hbar c}\psi - \frac{iq}{\hbar c}\vec{\nabla}(\Lambda)\, e^{iq\Lambda/\hbar c}\psi$$
$$= \vec{\nabla}(\psi)\, e^{iq\Lambda/\hbar c} + \psi\frac{iq}{\hbar c}e^{iq\Lambda/\hbar c}\vec{\nabla}(\Lambda) - \frac{iq}{\hbar c}\vec{A}e^{iq\Lambda/\hbar c}\psi - \frac{iq}{\hbar c}\vec{\nabla}(\Lambda)\, e^{iq\Lambda/\hbar c}\psi$$
$$= e^{iq\Lambda/\hbar c}\left(\vec{\nabla} - \frac{iq}{\hbar c}\vec{A}\right)\psi$$

$$\vec{D}'\psi' = e^{iq\Lambda/\hbar c}\vec{D}\psi$$

$$\vec{D}\cdot\vec{D}\psi = \vec{D}'\cdot\vec{D}'\psi' = e^{iq\Lambda/\hbar c}\vec{D}\cdot\vec{D}\psi$$

$$i\hbar D\psi = -\frac{\hbar^2}{2m}\vec{D}\cdot\vec{D}\psi$$

$$\to e^{iq\Lambda/\hbar c}\left(i\hbar D_0\psi\right) = e^{iq\Lambda/\hbar c}\left(-\frac{\hbar^2}{2m}\vec{D}\cdot\vec{D}\psi\right)$$

$$H = \frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + q\phi$$

Classical Mechanics: $\vec{F} = m\frac{d^2\vec{x}}{dt^2} = q\vec{E} + q\frac{\vec{v}}{c}\times\vec{B}$

$\vec{A}(\vec{x}(t),t)$, $\phi(\vec{x}(t),t)$ in H $\quad \vec{x}(t), \vec{p}(t)$

Hamilton's eq's: $\frac{\partial x_i}{\partial t} = \frac{\partial H}{\partial p_i}$, $\frac{\partial p_i}{\partial t} = -\frac{\partial H}{\partial x_i}$

$$\frac{\partial H}{\partial p_i} = \frac{\partial}{\partial p_i}\left(\frac{1}{2m}\left(p_j - \frac{q}{c}A_j\right)\left(p_j - \frac{q}{c}A_j\right)\right) = \frac{1}{m}\left(p_j - \frac{q}{c}A_j\right)\delta_{ij} = \frac{1}{m}\left(p_i - \frac{q}{c}A_i\right)$$

$$\frac{\partial H}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{1}{2m}\left(p_j - \frac{q}{c}A_j\right)^2 + q\phi\right)$$

$$= \frac{2}{2m}\left(p_j - \frac{q}{c}A_j\right)\left(\frac{\partial}{\partial x_i}\left(-\frac{q}{c}A_j\right)\right) + q\frac{\partial\phi}{\partial x_i}$$

$$= \frac{-q}{mc}\left(p_j - \frac{q}{c}A_j\right)\frac{\partial A_j}{\partial x_i} + q\frac{\partial\phi}{\partial x_i}$$

$$\frac{\partial x_i}{\partial t} = \frac{1}{m}\left(p_i - \frac{q}{c}A_i\right) \quad \leftarrow v_i = \frac{dx_i}{dt}$$

$$\frac{\partial p_i}{\partial t} = \frac{q}{mc}\left(p_j - \frac{q}{c}A_j\right)\frac{\partial A_j}{\partial x_i} - q\frac{\partial\phi}{\partial x_i}$$

$$\vec{p} = m\vec{v} - \frac{q}{c}\vec{A}$$

$$\frac{d\vec{x}}{dt} = \frac{\vec{p}}{m} \qquad \vec{p}(t)$$

$$m\frac{d^2\vec{x}}{dt^2} = m\frac{d}{dt}\left(m\vec{v} - \frac{q}{c}\vec{A}\right) = m\frac{dp_j}{dt} - \frac{mq}{c}\frac{d}{dt}\left(\vec{A}(\vec{x},t)\right)$$

$$= m\frac{dp_i}{dt} - \frac{mq}{c}\left(\frac{\partial A}{\partial t} + \frac{\partial A}{\partial x_i}\cdot\frac{dx_i}{dt}\right)$$

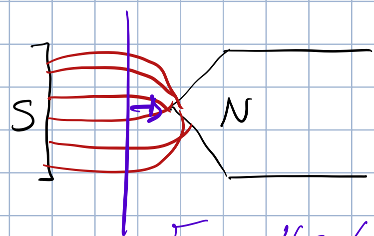
$$= \underbrace{q\left(\frac{-\partial\phi}{\partial x_i} - \frac{1}{c}\frac{\partial A_i}{\partial t}\right)}_{qE_i} \underbrace{- \frac{q}{c}\left(\frac{\partial A_i}{\partial x_k}\frac{\partial x_k}{\partial t}\right) + \frac{q}{c}\left(\frac{\partial x_i}{\partial t}\cdot\frac{\partial A_j}{\partial x_i}\right)}_{\frac{q}{c}\vec{v}\times\vec{B}}$$

check $\vec{\nabla} \times \vec{B} = \vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \frac{dx_i}{dt}\frac{\partial A_j}{\partial t} - \frac{dx_j}{dt}\frac{\partial A_i}{\partial x_j}$

Magnetic effects in matter are of 3 types

Ferromagnetism — magnetized iron — permanent magnetic moment

paramagnetic & diamagnetic



Force attracted to magnetic field

diamagnetic - repelled by a magnetic field

Constant magnetic fields, & compute change in energy in QM using perturbation theory ∴ (scaled)

Consider $\vec{E} = 0 \quad \vec{B} = B_0 \hat{e}_z$

can choose $\phi = 0$ & $\vec{A}(\vec{x})$, indpt of time b/c B not fn of time

can always make gauge transformations. 2 common ones

① symmetric gauge

$A_x = -By/2$
$A_y = Bx/2$
$A_z = 0$

$\vec{\nabla} \times \vec{A} = B$

② Landau gauge

$A_x = -By$
$A_y = 0$
$A_z = 0$

Consider Hydrogen atom in a constant magnetic field

Coulomb potential

$$H = \frac{1}{2\mu}\left(\vec{p}^{\,2} + \frac{e}{c}(\vec{A}\cdot\vec{p} + \vec{p}\cdot\vec{A}) + \frac{e^2}{c^2}\vec{A}\cdot\vec{A}\right) - \frac{e^2}{r}$$

$q = -e$
ignoring proton for now in coupling to $\vec{A}$

$$H = H_0 + \underbrace{\frac{e}{2\mu c}(\vec{B}\times\vec{r}\cdot\vec{p})}_{\text{linear in } B_0} + \underbrace{\frac{e^2}{8\mu c^2}(\vec{B}\times\vec{r})(\vec{B}\times\vec{r})}_{\text{quadratic in } B_0}$$

H_0 - Hamiltonian for Hydrogen atom w/o $\vec{B}$

$\vec{A} = \frac{1}{2}(\vec{B}\times\vec{r})$

if B is small enough that new terms give small corrections to energy,
we can use perturbation theory

① Need to include spin

② B, B^2 terms in H

$\uparrow$

1st order