Sudden approximation: $H^{(1)}(t) \neq 0$ for $0 < t < \epsilon$, what happens as $\epsilon \to 0$?

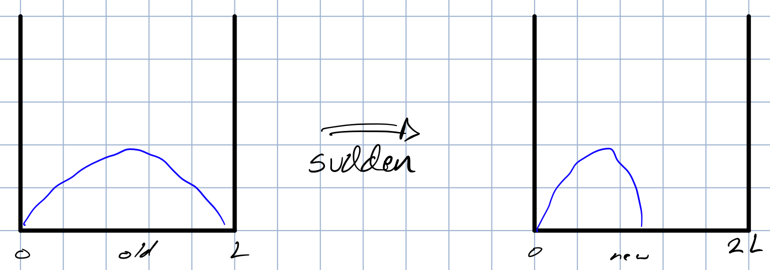
SE: $$\int_0^\epsilon i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle \, dt = \int_0^\epsilon \left(H^{(0)} + \lambda H^{(1)}(t)\right)|\psi(t)\rangle \, dt$$

$$i\hbar\left(|\psi(\epsilon)\rangle - |\psi(0)\rangle\right) = \int_0^\epsilon \left(H^{(0)} + \lambda H^{(1)}(t)\right)|\psi(t)\rangle \, dt$$

$|\psi(\epsilon)\rangle \sim |\psi(0)\rangle + \mathcal{O}(\epsilon)$

examples: tritium nucleus $\quad nnp \to \underbrace{ppn}_{He^3 \text{ nucleus}} + e^-$

what's prob electron in tritium atom is in grnd state of He^3 after decay?



1-D particle in box $\quad \psi^{old}_{grnd}(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) \qquad \psi^{new}_n(x) = \sqrt{\frac{1}{L}} \sin\left(\frac{n\pi x}{L}\right)$

if particle was in $\psi^{old}_{grnd}(x)$, what's prob. it's in grnd state of new box after size doubles?

$$\psi^{old}_{grnd}(x) = \sum_{n=1}^{\infty} c_n \psi^{new}_n(x)$$

$|c_1|^2 =$ prob. of being in new grnd state

$$\int_0^{2L} \psi^{new}_n(x)^* \psi^{new}_n(x) \, dx = \sum_{n=1}^{\infty} c_n \int_0^{2L} \psi^{new}_1(x)^* \psi^{new}_n(x)^* \, dx$$

$$= \sum c_n \delta_{n,1} = c_1$$

Harmonic Perturbations

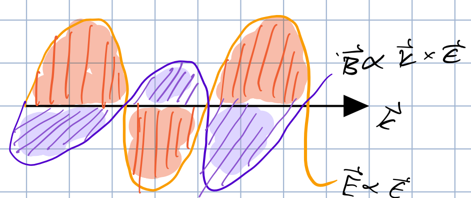
Maxwell: $\vec{B} = \vec{\nabla} \times \vec{A}$ $\qquad$ $\vec{E} = -\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$ $\qquad$ solve 2 eq^{n}s

Choice of gauge: $\vec{\nabla}\cdot\vec{A} = 0$, $\Phi = 0$ $\qquad$ (in vacuum $\rho = 0$ & $\vec{j} = \vec{0}$)

$$\Longrightarrow \left(-\vec{\nabla}\cdot\vec{\nabla} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A}(\vec{r},t) = 0$$

Sol^{n}: $\vec{A}(\vec{r},t) = \vec{\epsilon}\left(A_0 \exp[i(\vec{k}\cdot\vec{r} - \omega t)] + A_0 \exp[-i(\vec{k}\cdot\vec{r} - \omega t)]\right)$

E/M wave w/ polarization vector $\vec{\epsilon}_i$ ⊥ wave # $\vec{k}$



H-atom: $H = H^{(0)} + \frac{e}{m_e c}\vec{A}\cdot\vec{p} + \frac{e^2}{2mc^2}\vec{A}\cdot\vec{A}$

simple rule summarizing info from QED

Absorption of photon by an atom from state N (# of photons existing when starting absorption)

use $\vec{A}(\vec{r},t) = \left(\frac{2\pi c^2 N \hbar}{\omega V}\right)^{1/2} \vec{\epsilon}\, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$

V = volume of box, where we quantize E/M field

Emission of photon into state w/ $(N+1)$ photons, including emitted photon

$$\vec{A}(\vec{r},t) = \left(\frac{2\pi c^2 (N+1)\hbar}{\omega V}\right)^{1/2} \vec{\epsilon}\, e^{-i(\vec{k}\cdot\vec{r} - \omega t)}$$

Motivate & discuss N-dependence

quantized E/M field ≡ infinite # of SHO ~ $\vec{a}(\vec{k},\omega)$ $\qquad$ creation & annihilation operators

SHO: $\langle n-1 | a | n\rangle = \sqrt{n}$

$\langle n+1 | a^{+} | n\rangle = \sqrt{n+1}$

creating & destroying quantized energy (photons)

initial state: $|i\rangle$
final state: $|f\rangle \neq |i\rangle$

To 1st order in $H^{(1)}(t)$: $\qquad \omega_{fi} = (E_f - E_i)/\hbar$

last lecture → $$P_{i\to f} = \frac{1}{\hbar^2}\left| \int_0^t \langle f|H^{(1)}(t')|i\rangle\, e^{i\omega_{fi}t'}\, dt' \right.$$

Emitting a photon: $\vec{A} \neq 0$ for $N=0$

strength of interaction: $\sim \sqrt{N+1}$ — larger if there are photons in the state the emmitted photon is going into

stimulated emission – LASER!

For an atom emitting 1 photon

$$P_{i\to f} = \frac{1}{\hbar^2}\cdot\left(\frac{2\pi c^2\hbar}{\omega V}\right)\left(\frac{e}{m_e c}\right)^2 \left|\langle f|\vec{\epsilon}\cdot\vec{p}\cdot e^{-i\vec{k}\cdot\vec{r}}|i\rangle\right|^2 \left|\int_0^t e^{i(\omega_{fi}+\omega)t'}dt'\right|^2$$

with fixed polarization $\vec{\epsilon}$ & wave vector $\vec{k}$

more interested in transition rate than probability

① Analyze $\int dt'$. show it gives $P_{i\to f} \propto t$ & δ fn (related to energy conservation)

$$P_{i\to f} = \lim_{t\to\infty} \frac{P_{i\to f}}{t}$$

② Approximate & evaluate $\langle f| \quad\quad |i\rangle$

③ Sum over all decays: $\int d^3k$ & $\sum_{\epsilon}$