Most QM systems have natural time scale

SHO: $T \sim 1/\omega$

H atom: $T \sim \frac{\hbar}{E_{ground}}$

Perturbations can be divided into 3 cases

1. Adiabatic - time variation of perturbation is large compared to T
2. Sudden - time variation of perturbation is small compared to T
3. Harmonic - $H'(t) = a^{+} e^{i\omega t} + a e^{-i\omega t}$ cool when $\omega \sim 1/T$

assume $H = H^{(0)} + \lambda H^{(1)}(t)$ $\quad$ $H^{(0)}$ is indpt. of time $\quad$ $H^{(0)}|\phi_n\rangle = E_n|\phi_n\rangle$

want to solve SE $\quad i\hbar \frac{\partial}{\partial t}(|\Psi(t)\rangle) = \left(H^{(0)} + \lambda H^{(1)}(t)\right)|\Psi(t)\rangle$

$$|\Psi(t)\rangle = \sum_n \tilde{c}_n(t)\,|\phi_n\rangle = \sum_n \underset{\substack{\uparrow \\ \text{indpt. of} \\ \text{time when } \lambda H^{(1)}=0}}{c_n(t)}\, e^{-iE_n^{(0)}t/\hbar}\,|\phi_n\rangle$$

$$\sum_n \left(i\hbar \frac{d c_n(t)}{dt} + \cancel{E_n^{(0)}} c_n(t)\right) \cdot e^{-iE_n^{(0)}t/\hbar}|\phi_n\rangle = \sum_n \left(\cancel{E_n^{(0)}} + \lambda H^{(1)}(t)\right) c_n(t)\, e^{-iE_n^{(0)}t/\hbar}|\phi_n\rangle$$

$$\langle\phi_m|\, i\hbar \sum_n \frac{d c_n(t)}{dt} \cdot e^{-iE_n^{(0)}t/\hbar}|\phi_n\rangle = \langle\phi_m|\,\lambda \sum_n H^{(1)}(t)\, c_n(t)\, e^{-iE_n^{(0)}t/\hbar}|\phi_n\rangle$$

$$i\hbar \frac{d c_m(t)}{dt} = \lambda \sum_n c_n\, e^{it(E_m^{(0)} - E_n^{(0)})/\hbar} \langle\phi_m|H^{(1)}(t)|\phi_n\rangle$$

assume we can expand in λ $\quad c_n(t) = \underset{O(\lambda^0)}{c_n^{(0)}(t)} + \underset{O(\lambda)}{c_n^{(1)}(t)} + \cdots$

there's a λ on right side, get an eq^n for $c_n^{(1)}$ on left hand

@ time t_0 suppose start in state $|\phi_k\rangle$ of $H^{(0)}$

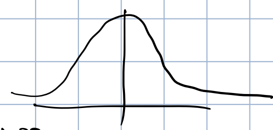
what's probability we are in state $|\phi_n\rangle$ @ later time t?

$c_n^{(0)} = \delta_{kn}$

$$\star\; c_n^{(1)}(t) = \frac{\lambda}{i\hbar}\int_{t_0}^{t} dt'\, e^{i(E_n^{(0)} - E_k^{(0)})t'/\hbar} \langle\phi_n|H^{(1)}(t')|\phi_m\rangle \;\star$$

example: SHO: $H^{(0)} = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2$

charge q & $\vec{E} = E\, e^{-(t/\tau)^2}\, \hat{e}_x$



$\underline{\lambda} H^{(1)}(t) = \underline{-e} E x e^{-(t/\tau)^2}$ $\qquad \vec{E} \to 0$ as $t \to \infty$

suppose $t_0 = -\infty$ $\quad c_n^{(0)}(-\infty) = \delta_{n,0}$

$E_n^{(0)} - E_0^{(0)} = (n+1/2)\hbar\omega - \frac{1}{2}\hbar\omega = n\hbar\omega$

$$\text{Prob}(0 \to n) = |c_n(t)|^2 = |c_n^{(0)} + c_n^{(1)}(t)|^2$$

$$= \left| \delta_{n,0} - \frac{i}{\hbar} \cdot \int_{-\infty}^{t} \underline{(-eE)} \langle n | x e^{-(t'/\tau)^2} | 0 \rangle e^{in\omega t'} dt' \right|^2$$

$$\langle n | x | 0 \rangle \propto \langle n | a + a^\dagger | 0 \rangle \Rightarrow \langle n | x | 0 \rangle = \sqrt{\frac{\hbar}{2m\omega}}\, \delta_{n,1}$$

$$\text{prob}(0 \to 1) = \frac{(eE)^2}{2m\omega\hbar} \left| \int_{-\infty}^{+\infty} e^{-(t'^2/\tau^2) + i\omega t'} dt' \right|^2 = \frac{e^2 E^2 \pi^2 \tau^2}{2m\omega\hbar} e^{-\omega^2\tau^2/2}$$

$$\text{prob}(0 \to 1) \sim \tau^2 e^{-\omega^2\tau^2/2}$$

τ - time scale of perturbation

$\to 0$ as $\tau \to 0$ $\qquad$ sudden

$\to 0$ as $\tau \to \infty$ $\qquad$ adiabatic (very slow)

largest @ $\omega \sim 1/\tau$