states $\quad \mathcal{O}|\psi\rangle$

operators: $\quad A \to \mathcal{O} A \mathcal{O}^{-1}$

$$K|\psi\rangle = |\psi\rangle^*$$
$$K\mathcal{O}K^{-1} = \mathcal{O}^*$$

2 loose ends including spin

Time reversal $\quad T: \quad \vec{L} = \vec{r}\times\vec{p} \longrightarrow -\vec{L} \quad \& \quad \vec{S} \to \vec{S}$

$$TST^{-1} = -\vec{S}$$

for spin 1/2: $\quad \vec{S} = \hbar/2\ \vec{\sigma}$

$$T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} K$$

unitary operator U on $\mathbb{R}^2$ — complex conjugation

$$T\sigma_x T^{-1} = U \quad K \quad \sigma_x \quad K^{-1} U$$
$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} K \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} K^{-1} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^* \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = -\sigma_x$$

$$T\sigma_x T^{-1} = -\sigma_x$$
$$T\sigma_y T^{-1} = -\sigma_y$$
$$T\sigma_z T^{-1} = -\sigma_z$$

$$K(\quad)K^{-1} = (\quad)^*$$

$$T^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} K^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \mathbb{1} = -\mathbb{1}$$

if H is a time reversal invariant Hamiltonian then $[H,T]=0$

$$H|x\rangle = E|x\rangle \longrightarrow HT|x\rangle = TH|x\rangle = TE|x\rangle = E\cdot T|x\rangle$$

$|x\rangle$ & $T|x\rangle$ have same energy

→ 2 possibilities:

① $|x\rangle, T|x\rangle$ are same state → $T|x\rangle = e^{i\phi}|x\rangle \quad (T^2=1)$

② $|x\rangle, T|x\rangle$ are distinct states w/same energy → $T^2=-1$

Okay take ① & assume $|\chi\rangle \neq$ zero vector

then $\underline{T|\chi\rangle = e^{i\phi}|\chi\rangle}$

$$\longrightarrow \quad T^2|\chi\rangle = Te^{i\phi}|\chi\rangle = \underbrace{Te^{i\phi}}_{\text{complex conjugate}}\overbrace{T^{-1}T}^{T^{-1}T=\mathbb{1}}|\chi\rangle = e^{-i\phi}\underline{T|\chi\rangle} = e^{-i\phi}\underline{e^{i\phi}|\chi\rangle} = |\chi\rangle$$

so if $T|\chi\rangle = e^{i\phi}|\chi\rangle$, then $T^2|\chi\rangle = |\chi\rangle$, or $T^2 = 1$

if $T^2 = -1 \longrightarrow T|\chi\rangle = -|\chi\rangle \longrightarrow$ so $|\chi\rangle$ is zero vector

any system $-N$ electrons, interacting $\vec{E} \neq 0$ & $\vec{B} = 0$

if $T^2 = (-1)$ (net spin ½)

$\longrightarrow$ 2 fold degeneracy in energy spectrum (Kramer's Thm)

act on spin up $TK|+\rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}^* = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|-\rangle$

What about effects of spin for an electron in a $\vec{B}$ field?

electrons have a magnetic moment: $\vec{\mu} = g\left(\frac{-e}{2m_e c}\right)\vec{S}$

$$H_{spin} = -\vec{\mu}\cdot\vec{B} = \frac{eB}{m_e c}\left(\frac{g}{2}\right)S_z$$

spin up & spin down have different energies

$$\Delta E_{\uparrow\downarrow} = \underline{\frac{eB\hbar}{m_e c}}\left(\frac{g}{2}\right)$$

$$\Delta E_{LLL} = E_{n+1} - E_n = \hbar\omega_B = \underline{\frac{eB\hbar}{m_e \cdot c}}$$ o'yo?

in vacuum, $g \approx 2 \longrightarrow \underline{\Delta E_{spin} = \Delta E_{LLL}}$

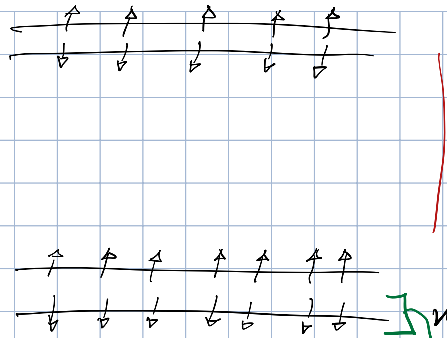
physics

MOSFET (GaAs)

Interactions change $m_e \to m_e^{eff}$ & $g \to g_{eff}$

$\Delta E_{spin} \sim \frac{1}{70}\Delta E_{LLL}$ (Zeeman effect)

*not relevant for materials



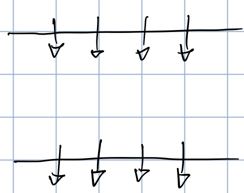
10x

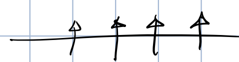
$n=0$ Landau level

lowest energy

math

$g \equiv 2$



$n=1$ or  $n=0$

$n=0$

2 states w/same energy $\rightarrow$ degeneracy. why?

degeneracy due to structure

$H = \{Q, Q\}$ $\quad [H, Q] = 0$

$n=1$ supersymmetry

Magnetic Monopoles

OG Maxwell's eqn's in vacuum

$$\left.\begin{aligned} \vec{\nabla} \cdot \vec{B} &= 0 \\ \vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} &= 0 \\ \vec{\nabla} \cdot \vec{E} &= 0 \\ \vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} &= 0 \end{aligned}\right\} \text{Symmetry}$$

D: $\vec{E} \rightarrow \vec{B}$, $\vec{B} \rightarrow -\vec{E}$ duality

D^2: $\vec{E} \rightarrow -\vec{E}$, $\vec{B} \rightarrow -\vec{B}$

$\vec{\nabla} \cdot \vec{E} = 4\pi \rho_e$

$\vec{\nabla} \cdot \vec{B} = 4\pi \rho_m$ (0 usually)

if so, $\rho_e \rightarrow \rho_m$ & $\rho_m \rightarrow -\rho_e$

magnetic monopoles

Dirac, Wu & Yang ~ 1954

QM $\Rightarrow$ electric charge is quantized $\quad N_q = Ne$

$\hookrightarrow N \in \mathbb{Z}$ integer