Last time: $\vec{B} = B_0 \hat{e}_z$, gauge $A_x = 0$ & $A_y = Bx$

SE sol$^{\underline{n}}$s: $\psi_n^{(k)} = e^{iky} U_n(x - x_0(k))$

$\uparrow$ n^{th} SHO eigenstates

$x_0(k) = \frac{\hbar c k}{qB}$

$\omega_B = \frac{|qB|}{m_e c}$

$P_y \, \psi_n^{(k)}(x,y) = \hbar k \, \psi_n^{(k)}(x,y)$

$H \psi_n^{(k)}(x,y) = (n + 1/2)\hbar\omega_B \, \psi_n^{(k)}(x,y)$

$U_0(x - x_0(k)) = \left(\frac{m\omega_B}{\pi\hbar}\right)^{1/4} e^{-m\omega_B(x - x_0(k))^2/2\hbar}$

$U_n(x - x_0(k)) = H_n(x - x_0) \, e^{-(\cdots)}$

$\uparrow$ nth order polynomial

$n = 0$ LLL

– $|\psi_n^k(x,y)|^2$ – indpt. of y

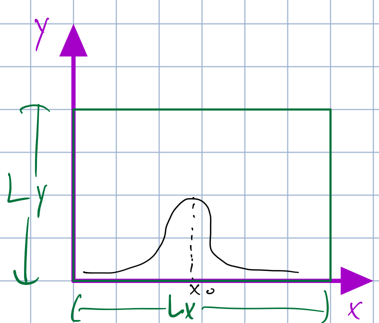
localized in x around x_0 w/ width $\sqrt{\frac{\hbar}{m\omega_B}}$

However, any linear combination of states in LLL has energy $\frac{1}{2}\hbar\omega_B$

$$\int_{-\infty}^{+\infty} e^{-\frac{sk^2}{2}} \, \psi_0^{(k)}(x,y) \, dk = \psi_{loc}(x,y) \quad \text{w/ } E = \tfrac{1}{2}\hbar\omega_B$$

This is for infinite systems – Infinite extent in x-y plane

In reality, we have finite systems



area $L_x \cdot L_y = A$

restrict $0 \le x_0 \le L_x$ & k

wave f$^{\underline{n}}$ can just be shifted

in y, we need boundary conditions

usually impose boundary conditions in y: $\psi_n^{(k)}(x,0) = \psi_n^{(k)}(x, L_y)$

$\psi_n^{(k)} = e^{iky} U_n(x - x_0) \Longrightarrow e^{ikL_y} = e^{ik\cdot 0} = 1 \rightarrow k = \frac{2\pi m}{L_y}$

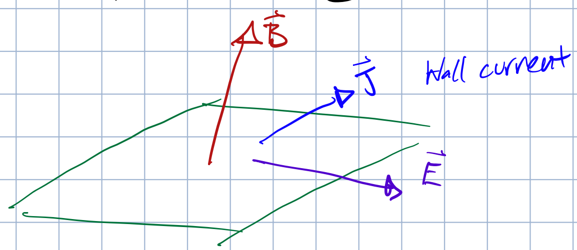
Using $x_0(k)$:

$\frac{qBx_0L_y}{\hbar c} = 2\pi m \Rightarrow 0 \le m \le \frac{qBL_xL_y}{2\pi\hbar c} = \frac{qBL_xL_y}{hc}$

for finite system given A, there's finite degeneracy labelled by m s.t. $0 \le m \le \frac{qB}{hc} \cdot A$

Now $\psi_n^{(m)}(x,y) = e^{\frac{2\pi i m y}{L_y}} U_n\left(x - x_0\left(\frac{2\pi m}{L_y}\right)\right)$ $\qquad m = 0, 1, 2, \ldots, m_{max}$

$$m_{max} = \frac{|qB|}{hc} \cdot A$$



$\vec{E} = E_y \vec{e}_y$, find $\vec{J} = J_x \vec{e}_x$

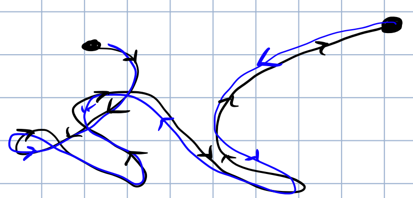
$\frac{J_x}{E_y} \equiv \sigma_{xy} = -f \frac{e^2}{h}$ $\qquad f$ - "filling fraction"

$$f = \frac{n_e}{n_{max}}$$

$n_e = \frac{\# \text{electrons}}{A}$ $\qquad n_{max} = \frac{m_{max}}{A}$

What about time reversal symmetry?

Classical Mechanics $\qquad (\vec{x}(t), \vec{p}(t)) \quad \mathcal{H}$



forward backward

$\vec{p} = m \cdot \frac{d\vec{x}}{dt}$

$T: \quad (\vec{x}(t), \vec{p}(t)) \to (\vec{x}(t), -\vec{p}(t))$

both are solutions to equations of motion

$T: \quad \vec{L} = \vec{x} \times \vec{p} \quad \to \quad -\vec{L}(-t)$

chang sign for angular momentum just like OG momentum

What about in QM?

Something that doesn't work

$x_{cl}(t) \sim \langle \psi(t) | \hat{x} | \psi(t) \rangle$

$p_{cl}(t) \sim \langle \psi(t) | \hat{p} | \psi(t) \rangle$ $\qquad$ expectation values of operators

so why not $\psi(t) \to \psi(-t)$, $\hat{x} \to \hat{x}$, $\hat{p} \to -\hat{p}$

$x_{cl} \to -x_{cl}$

$p_{cl} \to -p_{cl}$ $\qquad$ gang

But, $[\hat{x}, \hat{p}] = i\hbar$ $\qquad$ ∴ time reversal doesn't keep commutation fixed, but needs to!

E. Wigner 19?? classified types of symmetries in QM

① Unitary transformations – parity, translations, rotations <3

$$\Psi(t) \longrightarrow U\Psi(t) \quad , \text{ preserves } \quad |\langle x|\Psi\rangle|^2$$

② Always involves time reversal, anti-unitary operators

:o wow!

$$T = U K$$

unitary
often $U=\mathbb{1}$

complex conjugation $z = x+iy \to \bar{z} = x-iy$

$$T(\alpha_1\Psi_1 + \alpha_2\Psi_2) = \alpha_1^* T(\Psi_1) + \alpha_2^* T(\Psi_2)$$

$$T(\Psi) = U(\Psi^*)$$

$$|\langle x|\Psi\rangle| = \left|\int x^*\Psi\,dx\right|$$

$$\xrightarrow[U=\mathbb{1}]{T} \left|\int x\,\Psi^*\,dx\right| = |\langle x|\Psi\rangle|$$

$$T\hat{x}T^{-1} = \hat{x}^* = \hat{x}$$

$$T\hat{p}T^{-1} = \hat{p}^* = i\hbar\frac{d}{dx} = -\hat{p}$$

$$T[\hat{x},\hat{p}]T^{-1} = T i\hbar T = -i\hbar = -[\hat{x},\hat{p}]$$

$$i\hbar\frac{\partial}{\partial t}(\Psi) = H\Psi(t)$$

$$T\left(i\hbar\frac{\partial\Psi}{\partial t}\right) \overset{?}{=} T(H\Psi(t))$$

can put here b/c $T^{-1}T = \mathbb{1}$

$$-i\hbar\frac{\partial\Psi^*}{\partial t} = THT^{-1}T\Psi(t)$$

$$= H^*\Psi^*(t)$$

if $H = H^*$, $\Psi^*(-t)$ obeys same eq^n as $\Psi(t)$

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2 - 2xp_yc + \frac{q^2B^2x^2}{c^2}\right)$$

$p_y^* = -p_y$, H is not real

implies H is not time invariant