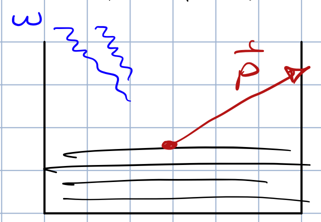
How Fermi's golden rule is applied depends on problem

Photoelectric effect



$|i\rangle$ - wave fn e^- in metal

$|f\rangle$ - plane wave

$$\langle \vec r | f \rangle = \frac{e^{i\vec p \cdot \vec r/\hbar}}{\sqrt{V}}$$

$$E_f = \frac{p^2}{2m} = E_i + \hbar\omega$$

$E_i < 0$

$\int d^3p = \int d\Omega \int p^2 dp$

$$\text{Rate} \sim \frac{2\pi}{\hbar} \int \frac{d^3p}{(2\pi\hbar)^3} \cdot \underbrace{|M_{fi}|^2}_{} \; \delta\underbrace{(E_f - E_i - \hbar\omega)}_{}$$

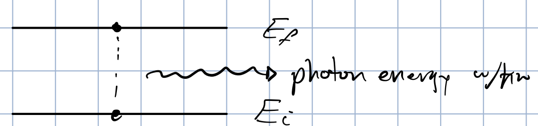
sum over all final e^- states

$\rightarrow E_f = E_i + \hbar\omega = \frac{\vec p^{\,2}}{2m}$

$M_{fi} = \langle f | V | i \rangle$ or $\langle f | V^\dagger | i \rangle$

$\lambda H^{(1)}(t) = V^\dagger e^{i\omega t} + V e^{-i\omega t}$

Atomic decay



QED says use $\frac{e}{m_e c} \vec A \cdot \vec p$ $\qquad \vec A = \left(\frac{2\pi c^2}{\hbar V}\right)^{1/2} \vec\epsilon \cdot e^{-i(\vec k \cdot \vec r - \omega t)}$

photon momentum

$$\text{Rate is} \quad \frac{2\pi}{\hbar} \int \frac{d^3p}{(2\pi\hbar)^3} \cdot |M_{fi}|^2 \cdot \delta(E_f - E_i + \hbar\omega^2)$$

For a photon decay of an excited atom

$$M_{fi} = \langle f | \frac{e}{m_e c} \vec A \cdot \vec p | i \rangle$$

$$\Gamma_{i\to f} = (\;) \cdot \int (\;) \cdot \underbrace{|\langle f | \vec\epsilon \cdot \vec p \, e^{-i\vec k \cdot \vec r} | i \rangle|^2}_{}$$

$\rightarrow$ if zero, decay rate is zero

$|i\rangle$ & $|f\rangle$ are Hydrogen wave fn's

$$\int \psi^*_{n_f,l_f,m_f}(\vec{r}) \cdot \vec{\epsilon}\,\vec{p}\, e^{-i\vec{k}\vec{r}}\, \psi_{n_i,l_i,m_i}(\vec{r})\, d^3\vec{r} = \langle f|(\ )|i\rangle$$

$K = |\vec{k}| = \omega/c$ photon energy: $E_\gamma = \hbar\omega$

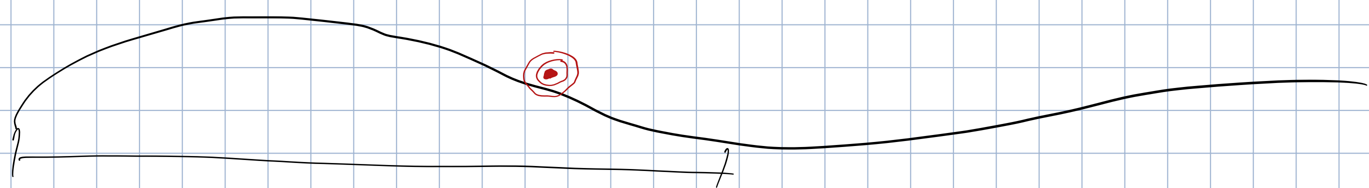
$$\langle i| e^{-i\vec{k}\vec{r}} |f\rangle = \langle i|(1 + i\vec{k}\vec{r} + \dots)|f\rangle$$

$\langle i|\ |\vec{r}|\ |f\rangle \sim a_0 = \frac{\hbar}{mc\alpha}$ $\alpha = 1/137$ expectation value of radius

Typical atomic energies are: $m_e c^2 \alpha^2$

$$\omega \sim \frac{m_e c^2 \alpha^2}{\hbar}$$

$$\langle f| \vec{k}\cdot\vec{r} |i\rangle \sim \frac{m_e c^2 \alpha^2}{m_e c^2 \alpha} = \alpha = 1/137$$



$\lambda_{photon} \sim 137$ atomic size

$$\langle f| e^{-i\vec{k}\vec{r}} \cdot \vec{\epsilon}\cdot\vec{p} |i\rangle \simeq \langle f|\vec{\epsilon}\cdot\vec{p}|i\rangle + O(\alpha)$$ electric dipole approximation

$$\langle f| \vec{\epsilon}\cdot\vec{p} |i\rangle = i m_e \omega_{fi} \vec{\epsilon}\, \underbrace{\langle f|\vec{r}|i\rangle}_{\text{electric dipole}}$$

$\vec{P}_{elec} = \int \vec{r} \rho_{elec}(\vec{r}) d^3\vec{r}$

ok, so **when is** $\vec{\epsilon}\cdot\langle f|\vec{r}|i\rangle \neq 0$?

$$\underset{\epsilon_x r_x + \epsilon_y r_y + \epsilon_z r_z}{\vec{\epsilon}\cdot\vec{r}} = r\sqrt{\frac{4\pi}{3}}\left(\epsilon_z Y_{1,0} + \frac{i\epsilon_y - \epsilon_x}{\sqrt{2}} Y_{1,1} + \frac{\epsilon_x + i\epsilon_y}{\sqrt{2}} Y_{1,-1}\right)$$

$\langle n_f, l_f, m_f|\ r\cdot Y_{1,m}\ | n_i, l_i, m_i\rangle$ $m \in [-1, 0, 1]$

angular integral is

$$e^{im_i\phi} \cdot P_{l_i}(\cos\theta)$$

$$\int \sin\theta d\theta \cdot \int d\phi \cdot Y^{*}_{l_F, m_F}(\theta,\phi) \cdot Y_{1,m}(\theta,\phi) \cdot Y_{l_i,m_i}(\theta,\phi)$$

ϕ integral $\rightarrow \int_0^{2\pi} d\phi \cdot e^{-im_F\phi} \cdot e^{im\phi} \cdot e^{im_i\phi} = \delta_{m+m_i-m_F,0}$

when $m_F = m + m_i$