① Schrödinger eq^n in 3-D

particle in a box

3-D SHO

spherical "square well"

H atom ← series sol^n

$$U(r) = r^{\alpha} \sum_{n=0}^{\infty} C_n r^n$$

indicial eq^n for α $r \to 0$ behavior of ODE

series sol^n for a 2nd order ODE w/ $r=0$ a regular singular point

② Continuous Symmetries in QM

$$U = e^{-i\alpha T}$$

unitary operator — real param — Hermitian operator

$$U|\psi\rangle = |\psi'\rangle$$

$$U|x\rangle = |x'\rangle$$

$$\langle x|\psi\rangle = \langle x|U^{+}U|\psi\rangle = \langle x'|\psi'\rangle$$

Symmetry if $[H, T] = 0$

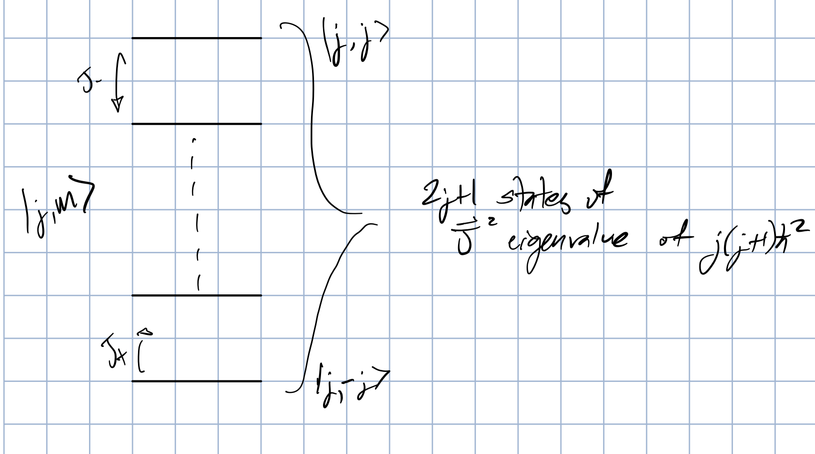
or $\underbrace{U^{+} H U}_{\text{expand to } O(\alpha)} = H$

$T = p_x, p_y, p_z$

$T = L_x, L_y, L_z$

$T = J_x, J_y, J_z$

$$[J_i, J_j] = i\hbar\, \epsilon_{ijk} J_k$$



$$J_\pm |j,m\rangle = \sqrt{j(j+1) - m(m\pm1)}\ |j, m\pm1\rangle$$

Sep. of variables $\quad L_i = \epsilon_{ijk}\, r_j\, p_k$

$$\psi(\vec{r}) = \underbrace{R_{nl}(r)\, Y_{lm}(\theta,\phi)}_{\text{basis in Hilbert space}}$$

$$f(\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty} C_{l,m}\, Y_{lm}(\theta,\phi)$$

$$\underbrace{\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta}_{d\Omega}\ Y^*_{l'm'}(\theta,\phi)\, Y_{lm}(\theta,\phi) = \delta_{ll'}\,\delta_{mm'}$$

$$\frac{1}{r}(x \pm iy) = \sin\theta\, e^{\pm i\phi} \propto Y_{1,\pm1}(\theta,\phi)$$

$$\frac{1}{r} z = \cos\theta \propto Y_{1,0}(\theta,\phi)$$

rotation about z axis

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

what are its eigenfunctions & eigenvalues of matrix for general θ?

$R(\theta)$ is 2×2 matrix w/ entries in $\mathbb{R}$

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 \\ \pm i \end{pmatrix} = \begin{pmatrix} e^{\pm i\theta} \\ \mp i e^{\pm i\theta} \end{pmatrix} = e^{\pm i\theta}\begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$