S-O spin orbit interaction

Fine structure of H-atom

$H_0 = \frac{p^2}{2\mu} - \frac{e^2}{r}$

$\lambda H_1^{rel} = -\frac{1}{8} \frac{p^4}{\mu^3 c^2}$

$\lambda H_1^{S-O} = \frac{1}{2 m_e^2 c^2} \frac{e^2}{r^3} \vec{S} \cdot \vec{L}$

$\vec{L} \cdot \vec{S} = \frac{1}{2} (\vec{J}^2 - \vec{L}^2 - \vec{S}^2)$

$l \otimes \frac{1}{2} = (l + \frac{1}{2}) \oplus (l - \frac{1}{2})$

$|n, l, m_l, s, m_s\rangle \rightarrow |n, j, m_j, l, s\rangle$

use $\langle nlm | \frac{1}{r^3} | nlm \rangle$ from last lecture

$\langle n, j, m_j, l, s | \vec{L} \cdot \vec{S} | n, j, m_j, l, s \rangle$

$= \frac{\hbar^2}{2} \left(j(j+1) - l(l+1) - s(s+1) \right)$, $s = \frac{1}{2}$, $j = l + \frac{1}{2},\ l - \frac{1}{2}$

$= \frac{\hbar^2}{2} \begin{cases} l & j = l + \frac{1}{2} \\ -(l+1) & j = l - \frac{1}{2} \end{cases}$

$\lambda E_{n,l}^{(1)\, S-O} = \frac{1}{4} \frac{m_e c^2 \alpha^4}{n^3 l(l+1)(l+\frac{1}{2})} \times \begin{cases} l & j = l + \frac{1}{2} \\ -(l+1) & j = l - \frac{1}{2} \end{cases}$

$\lambda E_{n,l}^{(1)\, fine} = -\frac{1}{2} m_e c^2 \alpha^4 \frac{1}{n^3} \left(\frac{1}{j + \frac{1}{2}} - \frac{3}{4n} \right) = \lambda E_{n,l}^{(1)\, relative} + \lambda E_{n,l}^{(1)\, S-O}$

Ground state: $E = -\frac{1}{2} m_e c^2 \alpha^2 \left(1 + \frac{\alpha^2}{4}\right)$

1st excited states $n = 2$ $l = 0, 1$

$n = 2$ $l = 0 \rightarrow j = \frac{1}{2} \rightarrow {}^2S_{1/2}$

$n = 2$ $l = 1 \rightarrow j = \frac{3}{2}$ or $\frac{1}{2} \rightarrow {}^2P_{3/2}$ ${}^2P_{1/2}$

${}^2S_{1/2}$, ${}^2P_{3/2}$, ${}^2P_{1/2}$ $\longrightarrow$

${}^2P_{3/2}$

${}^2P_{1/2}$ ${}^2S_{1/2}$

} fine structure

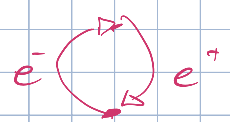
1947 → Lamb & Rutherford

$E({}^2S_{1/2}) - E({}^2P_{1/2}) = h\nu \qquad \nu \sim 10^3\,MHz$

Lamb shift → motivated development of QED (QFT of photons & electrons)

QFT $\quad E = mc^2$

$\Delta E\,\Delta t \geq \hbar$



electron - ½ spin

nucleus of H-atom is a proton - spin ½ & magnetic moment $\vec{\mu}_p = \gamma_p \vec{S}_p$

there's electron & proton magnetic moment: $\vec{\mu}_p \uparrow \quad \vec{\mu}_e \uparrow$ w/ energies different w/ spin

ground state $n=1, l=0, m_l=0$

$\vec{S} = \vec{S}_p + \vec{S}_e$

$$\langle 100 | H^{(1)\,\text{hyperfine}} | 100 \rangle = \frac{4}{3}\, g_p \frac{m_e^2}{m_p} \alpha^4 c^2 \underbrace{\frac{\vec{S}_e \cdot \vec{S}_p}{\hbar^2}}$$

m_p: supa small $\sim 2000 \cdot m_e$

$$\vec{S}_e \cdot \vec{S}_p = \frac{1}{2}\left(\vec{S}^2 - \vec{S}_p^{\,2} - \vec{S}_e^{\,2}\right)$$

$$s(s+1) - \tfrac{1}{2}\left(\tfrac{1}{2}+1\right) - \tfrac{1}{2}\left(\tfrac{1}{2}+1\right)$$

$s = 1, 0 \qquad s_e + s_p,\ s_e + s_p - 1, \dots, |s_e - s_p|$

$$E_{100}(s=1) - E_{100}(s=0) = h\nu = \frac{hc}{\lambda}$$

$\Leftrightarrow \nu = 1420\,MHz$

$\lambda = 21\,cm$

Practice Problem 5

Carbon $\underbrace{(1s)^2(2s)^2}_{l=0,\, s=0 \text{, closed shell}}(2p)^2$

$\vec{J}^2, \vec{L}^2, \vec{S}^2$ consistent w/ Fermi

can focus on $(2p)^2$

2 spin $s = 1/2$
2 $l = 1$ orbital wave f^c

$\vec{S} = \vec{S}_1 + \vec{S}_2$
$\vec{L} = \vec{L}_1 + \vec{L}_2$
$\vec{J} = \vec{L} + \vec{S}$

wave f^c for each electron

$\underbrace{R_{nl}(r)}_{\uparrow} Y_{lm}(\theta,\phi) \otimes |\pm\rangle$

radial part is symmetric

spin part

spin $\frac{1}{2} \otimes \frac{1}{2} = \underset{\text{anti}}{0} \oplus \underset{\text{sym}}{1}$

orbital $1 \otimes 1 = \underset{\text{sym}}{2} \oplus \underset{\text{anti}}{1} \oplus \underset{\text{sym}}{0}$

allowed (l,s) are $(2,0)$
$(0,0)$
$(1,1)$

j values
$2 \otimes 0 = 2$
$0 \otimes 0 = 0$
$1 \otimes 1 = 2 \oplus 1 \oplus 0$

s	l	j	
1	1	2	$^3P_2$
1	1	1	$^3P_1$
1	1	0	$^3P_0$

0	0	0	$^1S_0$
2	0	2	$^1D_2$