$$H = H_0 + \lambda H_1$$

$$H_0 |n\rangle = E_n^{(0)} |n\rangle$$

$$H |\psi_n\rangle = E_n |\psi_n\rangle$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)}$$

$$E_n^{(1)} = \langle n | H | n \rangle$$

(1) $E_n^{(2)}$ b/c sometimes $E_n^{(1)} = 0$
(2) how to deal w/ degenerate eigenvalues

Example:

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 + \lambda H_1$$

i) $\lambda H_1 = \lambda x^3$

$$E_n^{(1)} = \langle n | \lambda x^3 | n \rangle$$

$$= \int_{-\infty}^{+\infty} |\psi_n(x)|^2 x^3 dx = 0$$

even odd

$$x = \sqrt{\frac{\hbar}{2m\omega}} (a + a^+)$$

$$\langle n | (a + a^+)^3 | n \rangle$$

$$\langle n | a^3 + a^2 a^+ + a a^+ a^+ \dots | n \rangle = 0$$

ii) $\lambda H_1 = \lambda x^4$

$$E_n^{(1)} = \langle n | x^4 | n \rangle = \left(\frac{\hbar}{2m\omega}\right)^2 \langle n | (a + a^+)^4 | n \rangle$$

16 terms → only 6 terms where num of a^+ = num of a

$$\langle n | a a a^+ a^+ + a a^+ a a^+ + a^+ a a a^+ + a^+ a a^+ a + a^+ a^+ a a + a a^+ a^+ a | n \rangle$$

$$E_n^{(1)} = \frac{3\hbar^2}{4m^2\omega^2} (1 + 2n + 2n^2)$$

using $a|n\rangle = \sqrt{n}\,|n+1\rangle$ & $a^+|n\rangle = \sqrt{n+1}\,|n-1\rangle$

iii) $\lambda H_1 = \lambda x^2$

$$\frac{1}{2} m\omega^2 x^2 + \lambda x^2 = \frac{1}{2} m \omega'^2 x^2$$

$$\omega' = \omega^2 + \frac{2\lambda}{m}$$

$$E_n = \hbar\omega' (n + 1/2) = \omega \left(1 + \frac{\lambda}{m\omega} + O(\lambda^2)\right)$$

Atomic Systems often degenerate when we study tho

1^{st} order degenerate perturbation theory

$H_0|n\rangle = E_n^{(0)}|n\rangle$

$\hookrightarrow$ d_n states of energy $E_n^{(0)}$

$|n_i\rangle \quad i = 1, \dots, d_n$

give each degenerate state a specific α_i value or β_j value

$H_0|n,i\rangle = E_n^{(0)}|n,i\rangle$

$$|\psi_n(\lambda)\rangle = N(\lambda)\left[\sum_{i=1}^{d_n}\alpha_i|n,i\rangle + \sum_{k\neq n}\sum_{j=1}^{d_n}\beta_j\, C_{n,k}^{(0)}|k,j\rangle\right]$$

put into $(H_0+\lambda H_1)|\psi_n(\lambda)\rangle = (E_n^{(0)} + \lambda E_n^{(1)} + \dots)|\psi_n(\lambda)\rangle$

to $O(\lambda)$ is $\lambda H_1 \sum \alpha_i |n,i\rangle$ + stuff ortho. to $|n,j\rangle$

rhs has $\lambda E_n^{(1)} \sum \alpha_i |n,i\rangle$ + stuff ortho to $|n,i\rangle$

$\langle n,i|k,j\rangle = 0$

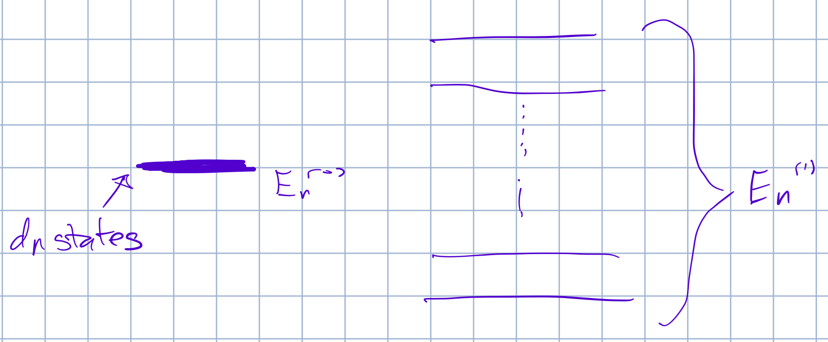
$$\langle n,j|\lambda H_1 \sum\alpha_i |n,i\rangle + 0 = \lambda E_n^{(1)}\langle n,j|\sum\alpha_i|n,i\rangle + 0$$

$$\sum_i \langle n,j|\lambda H_1|n,i\rangle\alpha_i = \lambda E_n^{(1)}\alpha_j$$

$$\sum_i (\lambda H_1)_{ji}^{(n)}\alpha_i = \lambda E_n^{(1)}\alpha_j$$

$$(\lambda H_1)\begin{pmatrix}\alpha_1\\ \vdots \\ \alpha_{d_n}\end{pmatrix} = \lambda E_n^{(1)}\begin{pmatrix}\alpha_1\\ \vdots \\ \alpha_{d_n}\end{pmatrix}$$

1^{st} order corrections to the energy of a d_n-fold deg. energy eigenstate are the eigenvalues of the perturbation in the degenerate space



Helium - using pert. theory for

grnd state — 1^{st} order pert. theory

1^{st} excited ~~state~~ — deg 1^{st} order

"derive" Hund's rules regarding spin

$$H = \underbrace{\frac{\vec{p}_1^{\,2}}{2m_e} - \frac{ze^2}{|\vec{r}_1|} + \frac{\vec{p}_2^{\,2}}{2m_e} - \frac{ze^2}{|\vec{r}_2|}}_{H_0} + \underbrace{\frac{e^2}{|\vec{r}_1 - \vec{r}_2|}}_{\lambda H_1}$$