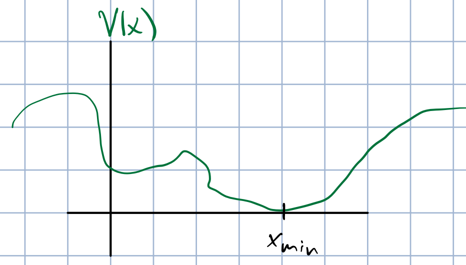
Uncertainty principle

$$\Delta x \, \Delta p \geq \hbar/2$$

$$H = \frac{\vec{p}^{\,2}}{2m} + V(x)$$



classically $p=0$, $x = x_{min}$, $E = 0$

in QM, best we can do

$$x = x_{min} + \Delta x$$

$$\Delta p \simeq \frac{\hbar}{2\Delta x}$$

1-D SHO

$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2$$

$$E(\Delta x) = \left(\frac{\hbar}{2\Delta x}\right)^2 \cdot \frac{1}{2m} + \frac{1}{2} m \omega^2 (\Delta x)^2$$

$$\frac{dE}{d\,\Delta x} = \frac{\hbar^2}{8m}\left(\frac{-2}{(\Delta x)^3}\right) + m\omega^2 \Delta x = 0$$

$$(\Delta x)^2_{min} = \frac{\hbar}{2m\omega}$$

$$(\Delta x)^4 = \frac{\hbar^2}{4m^2\omega^2}$$

Estimate for ground state E

$$E(\Delta x_{min}) = \frac{\hbar^2}{8m(\Delta x_{min})^2} + \frac{1}{2} m\omega^2 (\Delta x_{min})^2$$

$$= \frac{\hbar^2}{8m \, \frac{\hbar}{2m\omega}} + \frac{1}{2} m\omega^2 \left(\frac{\hbar}{2m\omega}\right)$$

$$= \frac{\hbar\omega}{4} + \frac{\hbar\omega}{4} \simeq \frac{\hbar\omega}{2}$$

gang

Pertur time

$H = H_0 + \lambda H_1$

① we know energy eigenvalues & eigenfunctions of H_0

② the term λH_1 is small "in some sense"

Goal: compute energies & eigenf's of H as power series in λ

for example: ① $H = \underbrace{\frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2}_{H_0} + \underbrace{\lambda x^4}_{\lambda H_1}$

② Helium atom

$$H = \underbrace{\frac{\vec{p}_1^{\,2}}{2m} + \frac{\vec{p}_2^{\,2}}{2m} - \frac{2e^2}{r_1} - \frac{2e^2}{r_2}}_{H_0} + \underbrace{\frac{e^2}{|\vec{r}_1 - \vec{r}_2|}}_{\lambda H_1}$$

③ Hydrogen atom

$$H = \underbrace{\frac{\vec{p}^{\,2}}{2\mu} - \frac{e^2}{r}}_{H_0} + \underbrace{\frac{1}{2m_e c^2}\frac{e^2}{r^3}\vec{S}\cdot\vec{L}}_{\lambda H_1}$$

$H_0 |n\rangle = E_n^{(0)} |n\rangle$

not SHO states

the $|n\rangle$ form a basis & we know them

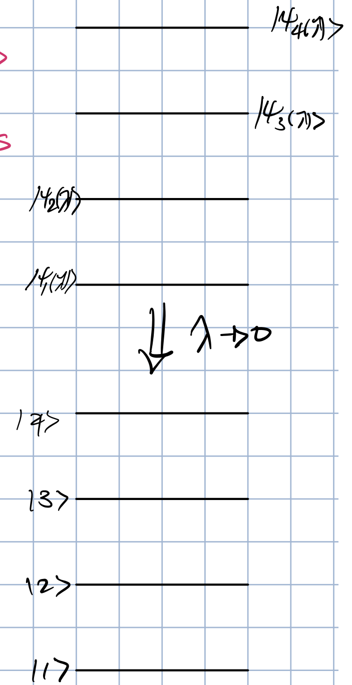
assume no degeneracy

$|\psi_n(\lambda)\rangle = \sum_k \tilde{c}_{nk}(\lambda) |k\rangle$

nth eigenf^{k} of full state

some coefficients

$H|\psi_n\rangle = E_n |\psi_n\rangle$



$H|\psi_n(\lambda)\rangle = E_n(\lambda)|\psi_n(\lambda)\rangle$

normalization constant

H_0 eigenstates

like picking out state ψ_n goes to as $\lambda \to 0$

$$|\psi_n(\lambda)\rangle = N(\lambda)\left[|n\rangle + \sum_{k\neq n} C_{n,k}(\lambda)|k\rangle\right]$$

$N(0) = 1$, $C_{n,k}(0) = 0$, $E_n(0) = E_n^{(0)}$

order

③ $E_n(\lambda) = E_n^{(0)} + \lambda E_n^{(1)} + \lambda^2 E_n^{(2)} + \ldots$

① $C_{n,k}(\lambda) = \cancel{C_{n,k}(0)} + \lambda C_{n,k}^{(1)}(\lambda) + \lambda^2 C_{n,k}^{(2)}(\lambda) + \ldots$

② $N(\lambda) = N(0) + \lambda N^{(1)} + \lambda^2 N^{(2)} + \ldots$

Plug into SE.

$$H\,|\Psi_n(\lambda)\rangle = (H_0 + \lambda H_1)\,|\Psi_n(\lambda)\rangle = E_n(\lambda)\,|\Psi_n(\lambda)\rangle$$

$$(H_0+\lambda H_1)\overbrace{(1+\lambda N^{(1)}+\dots)}^{2}\overbrace{\left(|n\rangle + \sum_{n,k} C_{n,k}^{(1)}(\lambda)\,|k\rangle + \dots\right)}^{1} = \overbrace{(E_n^{(0)} + \lambda E_n^{(1)} + \dots)}^{3}\overbrace{\left(|n\rangle + \sum_{n,k} C_{n,k}^{(1)}(\lambda)\,|k\rangle + \dots\right)}^{1}$$

λ^0 : $H_0|n\rangle = E_n^{(0)}\,|n\rangle$

λ^1 : $\lambda H_1|n\rangle + \lambda \sum_{k\neq n} C_{n,k}^{(1)} \underbrace{H_0|k\rangle}_{\to E_0|k\rangle} = \lambda E_n^{(1)}\,|n\rangle + \lambda E_n^{(0)} \sum_{k\neq n} C_{n,k}^{(1)}\,|k\rangle$

$\langle n|n\rangle \to 0$ $\langle n|k\rangle = \delta_{nk}$ but $k\neq n$

apply $\langle n|$ & use $\langle n|k\rangle = \delta_{nk}$

lhs : $\lambda\langle n|H_1|n\rangle = \lambda E_n^{(1)}\langle n|n\rangle$

$\lambda\langle n|H_1|n\rangle = \lambda E_n^{(1)}$

The $O(\lambda)$ correction to the energy of state $|n\rangle$

is the expectation value of perturbation (λH_1) in state $|n\rangle$

Oopsies @ degeneracy states or