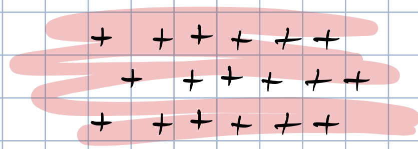
Many N spin ½ particles confined to a region w/volume V

piece of silver: electron config. Kr $(4d)^{10}$ $(5s)^1$
$\ell=2$

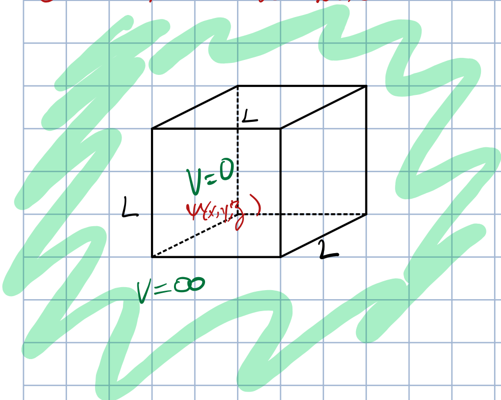
+ - silver ion w/o 5s electrons
electron wave fn's



approximations: electrons are confined within the silver & don't interact w/ions or each other

also: electrons in a white dwarf star
neutrons in a neutron star

Start w/a cube box



$$\psi_{n_x,n_y,n_z}(x,y,z) = \sqrt{\frac{8}{L^3}}\ \sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)\sin\left(\frac{n_z\pi z}{L}\right)$$

$n_x, n_y, n_z = 1, 2, \ldots$

$$E_{n_x,n_y,n_z} = \frac{\hbar^2\pi^2}{2mL^2}\left(n_x^2+n_y^2+n_z^2\right)$$

lowest energy state? w/large N

$N=1$ $\psi_{111}(\vec{x}) \otimes |\pm\rangle$

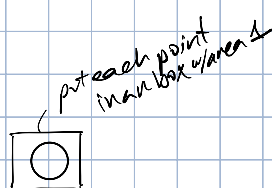
$N=2$ $\psi_{111}(\vec{x}) \otimes |+\rangle,\ \psi_{111}(\vec{x}) \otimes |-\rangle$

$N=3$ $\psi_{211},\ \psi_{111},\ \psi_{111}$

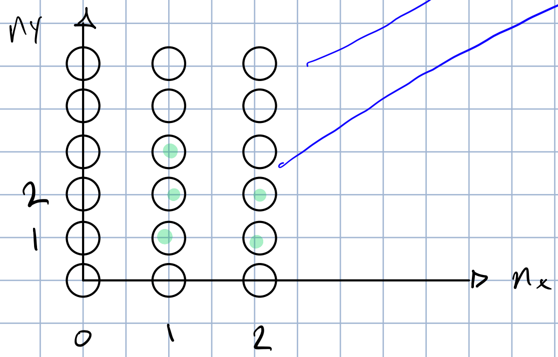
⋮

$N = 10^{23}$

what is highest energy occupied state?
what is the total energy?
functions of N, V: $n = N/V$

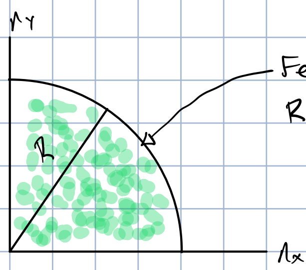


$\vec{n} = (n_x, n_y, n_z)$



$E \propto (n_x^2 + n_y^2)$

make as close to origin as possible



Fermi energy

$R^2 = n_x^2 + n_y^2 + n_z^2$ of last electron we add

$\simeq \frac{2mL^2}{\hbar^2\pi^2} E_f$

how many points are there in the positive octant of a sphere radius R?

$\frac{1}{8} Vol(R) = \frac{1}{8}\left(\frac{4}{3}\pi R^3\right)$

spin of electrons takes care of one particle state

Total # of electrons $\quad 2 \times \frac{1}{8} Vol(R)$

$$N = \frac{\pi}{3}\left(\frac{2mL^2 E_f}{\hbar^2\pi^2}\right)^{3/2}$$

with $R^2 =$

$n = N/L^3$ num density

$$E_f = \frac{\hbar^2\pi^2}{2m}\left(\frac{3N}{\pi L^3}\right)^{2/3} = \frac{\hbar^2\pi^2}{2m}\left(\frac{3n}{\pi}\right)^{2/3}$$

spin :)

Total energy is $E_{Tot} = \sum_{n_x, n_y, n_z} \frac{\hbar^2\pi^2}{2mL^2}(n_x^2 + n_y^2 + n_z^2) \cdot 2 \longrightarrow$:o $\int$

s.t. $n_x^2 + n_y^2 + n_z^2 \le R^2$

& $n_x, n_y, n_z > 0$

spin

$$E_{Tot} \simeq 2 \cdot \frac{1}{8} \int d^3n \, \frac{\hbar^2\pi^2}{2mL^2} \vec{n}\cdot\vec{n}$$

positive octant

$$= 2 \cdot \frac{1}{8} \, 4\pi \, \frac{\hbar^2\pi^2}{2mL^2} \int_0^R n^2 n^2 \, dn$$

$\int d\Omega$ $\qquad \frac{1}{5}R^5$

volume

$$E_{Tot} = \frac{\hbar^2\pi^3}{10m}\left(\frac{3n}{\pi}\right)^{5/3} \cdot V$$

$n = N/V$

$$E_{Tot} \propto V^{-5/3} \cdot V = V^{-2/3}$$

$$P_{degeneracy} = - \left. \frac{\partial E_{Tot}}{\partial V} \right|_{N_{fixed}} \neq 0 \longrightarrow \text{degeneracy pressure}$$

bulk modulus

solids hard b/c of degeneracy pressure. aren't moving to higher energy states