Office Hours in person 5-6pm Th.

$$H = \frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2}\overbrace{(x^2+y^2+z^2)}^{r}$$

$$\frac{m}{2}\left(\omega_x^2 x^2 + \omega_y y^2 + \omega_z z^2\right)$$

$$H\,\phi = E\,\phi$$

$$\phi(x,y,z) = \phi(x)\,\phi(y)\,\phi(z)$$

subst. into SE & divide by ϕ

$$-\frac{\hbar^2}{2m}\frac{d^2\phi_1}{dx^2} + \frac{1}{2}m\omega^2 x^2\,\phi_1(x) = E_1\,\phi_1(x)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\phi_1}{dy^2} + \frac{1}{2}m\omega^2 y^2\,\phi_2(y) = E_2\,\phi_2(y)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\phi_1}{dz^2} + \frac{1}{2}m\omega^2 z^2\,\phi(z) = E\,\phi_3(z)$$

$$E = E_1 + E_2 + E_3$$

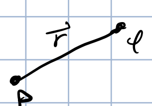
$$E_1 = (n_x + \tfrac{1}{2})\hbar\omega$$
$$E_2 = (n_y + \tfrac{1}{2})\hbar\omega$$
$$E_3 = (n_z + \tfrac{1}{2})\hbar\omega$$

ground state : $n_x = n_y = n_z = 0$

$$E = \frac{3}{2}\hbar\omega$$

3 1st excited state: $(n_x, n_y, n_z) = (1,0,0),\ (0,1,0),\ (0,0,1)$

$$E = (\tfrac{3}{2} + 1)\hbar\omega$$



3 H-atom

$$r = \sqrt{x^2+y^2+z^2}$$

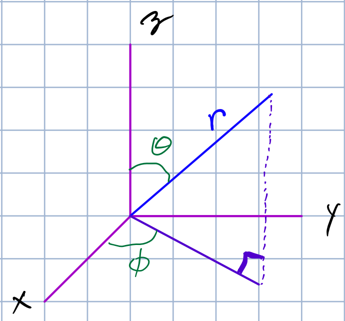
$$H_{rel.} = \frac{\vec{p}^2}{2\mu} - \frac{e^2}{r}$$

$$\phi(\vec{r}) = R(r)\,Y(\theta,\phi)$$

can't solve by Sep. of Vars. in cartesian

$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$

$\theta = [0, \pi)$
$\phi = [0, 2\pi)$



For continuous symmetry transformations

$$U(\alpha) = \exp(-i\alpha t) = 1 - i\alpha t + \frac{1}{2!}(-i\alpha)^2 t^2 + \dots$$

unitary operator; parameter labels the transformation, $\mathbb{R}$; hermitian operator

$$U^{+} = (\exp(-i\alpha t))^{+} = \exp(i\alpha t^{+}) = \exp(i\alpha t)$$

$$U^{+}U = e^{i\alpha t} \cdot e^{-i\alpha t} = \mathbb{1}$$

All states $|\psi\rangle \in \mathcal{H}$, symmetry transformation $|\psi\rangle \rightarrow U(\alpha)\,|\psi\rangle$

if $[H, t] = 0$, we say H is invariant under symmetry transformation w/generator t

$$\frac{d}{dt}\langle\psi(t)|\,t\,|\psi(t)\rangle = \frac{-i}{\hbar}\langle\psi(t)|\,\overset{0}{[H,t]}\,|\psi(t)\rangle = 0$$

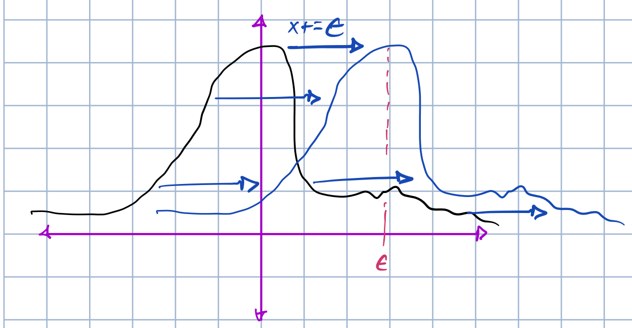
Ehrenfest's Thm

conserved quantity $\rightarrow$ exp value of t

Math: $U(\alpha)$ - element of Lie group
t - element of Lie algebra

1^{st} example: translation in 1-D

wave fn $\langle x | \phi \rangle = \phi(x)$



$\phi(x)$ "particle near $x=0$"

$\phi(x-\epsilon)$ "particle near $x=\epsilon$"

if $\phi(x)$ peaks @ $x=0$, this peaks @ ϵ

$$\phi(x-\epsilon) = \phi(x) - \epsilon \frac{d\phi}{dx} + \frac{(-\epsilon)^2}{2!} \frac{d^2\phi}{dx^2} + \dots = \sum \frac{(-\epsilon)^n}{n!} \frac{d^n \phi(x)}{dx^n}$$

$$= \exp\left(-\epsilon \frac{d}{dx}\right) \phi(x)$$

$$= \exp\left(-i\epsilon \frac{p}{\hbar}\right) \phi(x)$$

$p = -i\hbar \frac{d}{dx}$

$\frac{d}{dx} = -i\frac{p}{\hbar}$

$$\phi(x-\epsilon) = T(\epsilon)\,\phi(x)$$

$$T(\epsilon) = e^{-i\epsilon p/\hbar} = e^{(-i)\left(\frac{\epsilon}{\hbar}\right)(p)}$$

α t

$$U(\alpha) = e^{-i\alpha t}$$

$\alpha \sim \frac{\epsilon}{\hbar}$

$t = p$

$U(\alpha) = T(\epsilon)$

p is generator of translations

$\rightarrow$ if $[H, p] = 0$ then $\langle \psi(t) | p | \psi(t) \rangle$ is indpt. of time

$H = \frac{p^2}{2m} + U(x)$

$[H, p] = i\hbar \frac{dV}{dx} = 0$ if V is const if $[H, p] = 0$ $\quad |\psi\rangle \to U(\alpha)|\psi\rangle \quad \langle x|H|\psi\rangle$ are invariant under translations

$\langle x| H|\psi\rangle \to \langle x| T^{\dagger}(\epsilon)\, H\, T(\epsilon)|\psi\rangle$ inv. if $T^{\dagger}(\epsilon) H T(\epsilon) = H$

$e^{\frac{i\epsilon p}{\hbar}} H e^{-\frac{i\epsilon p}{\hbar}} = H$ $\quad$ pset 6 $\quad e^{-A} B e^{A}$

$T(\epsilon_1) T(\epsilon_2) = T(\epsilon_1 + \epsilon_2)$

$T(\epsilon \cdot - \epsilon) = T(0) = \mathbb{1}$

$T(\epsilon)$ form a group

$\to$ group is Abelian

in 3-D translation $\vec{\epsilon} = (\epsilon_x, \epsilon_y, \epsilon_z)$

$T(\vec{\epsilon}) = e^{-i\,\vec{\epsilon}\cdot\vec{p}/\hbar}$

$[p_x, p_y] = [p_x, p_z] = 0$