3 topics

a) QM in 3-D   focus on atomic systems
use series sol$^{\underline{n}}$'s of SHO

b) Exploit & understand symmetry in QM. focus on rotational systems
ladder structure for angular momentum
new QM element: spin ½
addition of angular momentum uses heavily tensor product construction
apply to atomic systems

c) Approx. methods: dimensional analysis, uncertainty principle, variational method, ~> perturbation theory

D=3

$$H = \frac{\vec{p}^{\,2}}{2m} + V(\vec{x})$$

$$\vec{p} = -i\hbar\vec{\nabla} = -i\hbar\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = (p_x, p_y, p_z) = (p_1, p_2, p_3)$$

$$\vec{x} = (x, y, z) = \vec{r} = (x_1, x_2, x_3)$$

$$[x_i, p_j] = i\hbar\,\delta_{ij}$$

energy eigenstates: $H\,\Phi_E(x) = E\,\Phi_E(x)$

$$\Psi_E(t, \vec{x}) = e^{-iEt/\hbar}\,\Phi_E(\vec{x})$$

$$\sum_E C_E\,\Phi_E(\vec{x}) \longrightarrow \sum_E C_E\,e^{-\frac{itE}{\hbar}}\,\Phi_E(\vec{x})$$

normalizable: $\underbrace{\int dx \int dy \int dz}_{\int d^3\vec{x} \,\to\, \int d^3r} |\Phi_E(\vec{x})|^2 = \text{finite}$

einstein summation convention

$$\vec{x}\cdot\vec{x} = \sum_i^3 x_i x_i$$

$$(\vec{x}\cdot\vec{x}, \vec{p}\cdot\vec{p}) = \sum_i x_i x_i \sum_j p_j p_j \equiv x_i x_i p_j p_j \neq x_i x_i p_i p_i$$

einstein

in $D=3$, SE

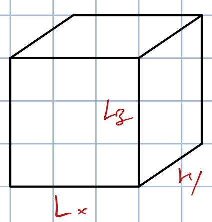
① is a PDE vs ODE

② requires using a coord. system adapted to symmetry of problem $(x,y,z) \to (r,\theta,\phi)$

③ sol^{n}'s involve new, special f^{n}'s

Bessel f^{n}'s, Laguerre poly, spherical harmonics

Particle in a box



$$V(x) = \begin{cases} 0 & \text{if } 0 \le x \le L_x,\ 0 \le y \le L_y,\ 0 \le z \le L_z \\ \infty \end{cases}$$

$$-\frac{\hbar^2}{2m}\vec{\nabla}\cdot\vec{\nabla}\,\phi_E(\vec{x}) = E\,\phi_E(\vec{x})$$

$\Phi_E(\text{sides of box}) = 0$

$\phi_E(0,y,z) = \phi_E(L_x,y,z) = 0$

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$$\phi_E(\vec{x}) = \phi_1(x)\,\phi_2(y)\,\phi_3(z)$$

$$\frac{-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\phi_1(x)\,\phi_2(y)\,\phi_3(z)}{\phi_1(x)\,\phi_2(y)\,\phi_3(z)} = \frac{E\,\phi_1(x)\,\phi_2(y)\,\phi_3(z)}{\phi_1(x)\,\phi_2(y)\,\phi_3(z)}$$

$$-\frac{\hbar^2}{2m}\left(\underbrace{\frac{1}{\phi_1(x)}\frac{\partial^2}{\partial x^2}}_{\text{f}^{n}\text{ of } x}+\underbrace{\frac{1}{\phi_2(y)}\frac{\partial^2}{\partial y^2}}_{\text{f}^{n}\text{ of } y}+\underbrace{\frac{1}{\phi_3(y)}\frac{\partial^2}{\partial z^2}}_{\text{f}^{n}\text{ of } z}\right) = \underbrace{E}_{\text{constant}}$$

$$-\frac{\hbar^2}{2m}\frac{1}{\phi_1}\frac{d^2\phi_1}{dx^2}=E_1$$

$$-\frac{\hbar^2}{2m}\frac{1}{\phi_2}\frac{d^2\phi}{dy^2}=E_2$$

$$E=E_1+E_2+E_3$$

$$-\frac{\hbar^2}{2m}\frac{1}{\phi_3}\frac{d^2\phi}{dz^2}=E_3$$

$$\phi_1(x)=C_1\sin\left(\frac{n_x\pi x}{L}\right) \qquad n_x=1,2,3,\ldots$$

$$\phi_2(y)=C_2\sin\left(\frac{n_y\pi y}{L}\right) \qquad n_y=1,2,3,\ldots$$

$$\phi_3(z)=C_3\sin\left(\frac{n_z\pi z}{L}\right) \qquad n_z=1,2,3\ldots$$

$$E=\frac{\hbar^2\pi^2}{2m}\left(\frac{n_x^2}{L_x^2}+\frac{n_y^2}{L_y^2}+\frac{n_z^2}{L_z^2}\right)$$

$$\phi_E(\vec{x})=\phi_{n_x}(x)\cdot\phi_{n_y}(y)\cdot\phi_{n_z}(z)$$

$$\chi(\vec{x})=\sum_{n_x n_y n_z} C_{n_x n_y n_z}\,\phi_E^{n_x n_y n_z}(\vec{x})$$