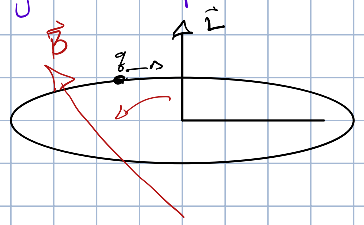
Some physical effects associated w/spin of electrons & of nuclei

magnetic dipole moment



$$\vec{\mu} = \frac{q\vec{L}}{2mc} \qquad \text{energy: } -\vec{\mu}\cdot\vec{B}$$

In QM, electrons/protons charge q, spin $\vec{S}$

$$\vec{\mu} = \gamma \vec{S}$$

$$\gamma = \frac{q}{2mc}\cdot g \quad \text{— g factor varies particle}$$

$\alpha = \frac{e^2}{\hbar c} \sim 1/137$

electron: $q = -e \qquad g = 2\left(1 + \frac{\alpha}{2\pi} + \dots\right)$

$$H = -\vec{\mu}\cdot\vec{B} = -\gamma\vec{S}\cdot\vec{B}$$

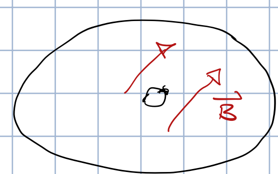
$$H_{real} = \underbrace{\frac{\vec{p}^2}{2m} + V(r)}_{\text{ignore for now}} - \gamma\vec{S}\cdot\vec{B}$$

Two state system

$\vec{S} = \frac{\hbar}{2}\sigma$ acts on $\mathbb{C}^2$

$\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$ are basis of S_z eigenstates w/eigenvalues $\pm\hbar/2$

$$H = -\gamma\vec{S}\cdot\vec{B}$$



choose z axis along $\vec{B}$

$$\vec{B} = B_0\,\hat{e}_z$$

$$H = -\gamma B_0 \frac{\hbar}{2}\sigma_z$$

$$= -\gamma B_0\frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Define $\omega_0 = \gamma B_0$

$$H = -\frac{\omega_0\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$\chi_+ = \begin{pmatrix}1\\0\end{pmatrix}$ spin up $\qquad E_+ = -\omega_0\,\hbar/2$

$\chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $E_- = +\omega_0\hbar/2$

at time $t=0$, spinor $\chi = \begin{pmatrix} a \\ b \end{pmatrix}$

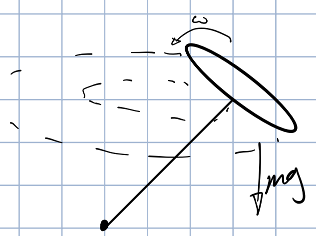
$\chi^\dagger \chi = |a|^2 + |b|^2 = 1$

$i\hbar \frac{\partial H}{\partial t} = H\chi$

$$\chi(t) = \begin{pmatrix} a \\ 0 \end{pmatrix} e^{-iE_+ t/\hbar} + \begin{pmatrix} 0 \\ b \end{pmatrix} e^{-iE_- t/\hbar}$$

$$= \begin{pmatrix} a e^{i\omega_0 t/2} \\ b e^{-i\omega_0 t/2} \end{pmatrix}$$

physically, this is precession of spin

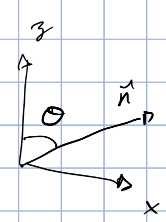


$a^2 + b^2$ $a = \cos\theta_0$ $b = \sin\theta_0$

@ $t=0$, $\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \cos(\theta_0) \\ \sin(\theta_0) \end{pmatrix}$

claim: $\hat{n} = (\sin\theta_0, 0, \cos\theta_0)$

$\chi(t=0)$ is eigenstate of $\hat{n} \cdot \vec{S}$



$$\hat{n} \cdot \vec{\sigma} = \sin\theta_0 \sigma_1 + \cos\theta_0 \sigma_3$$

$$= \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ \sin\theta_0 & -\cos\theta_0 \end{pmatrix}$$

check: $\begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ \sin\theta_0 & -\cos\theta_0 \end{pmatrix} \begin{pmatrix} \cos\theta_0/2 \\ \sin\theta_0/2 \end{pmatrix} = \begin{pmatrix} \cos(\theta_0)\cos(\theta_0/2) + \sin(\theta_0)\sin(\theta_0/2) \\ \sin(\theta_0)\cos(\theta_0/2) - \cos(\theta_0)\cos(\theta_0/2) \end{pmatrix} = \begin{pmatrix} \cos(\theta_0 - \theta_0/2) \\ \sin(\theta_0 - \theta_0/2) \end{pmatrix} = \begin{pmatrix} \cos\theta_0/2 \\ \sin\theta_0/2 \end{pmatrix}$

$\begin{pmatrix} \cos\theta_0/2 \\ \sin\theta_0/2 \end{pmatrix}$ is eigenstate of $\hat{n} \cdot \vec{S}$ w/ eigenvalue $+\hbar/2$

$$\chi(t) = \begin{pmatrix} \cos\theta_0/2 \; e^{i\omega_0 t/2} \\ \sin\theta_0/2 \; e^{-i\omega_0 t/2} \end{pmatrix}$$

exercise: show it is the eigenstate of $\hat{n}(t) \cdot \vec{S}$
w/ $\hat{n}(t) = (\sin(\theta_0)\cos(\omega_0 t), -\sin(\theta_0)\sin(\omega_0 t), \cos(\theta_0))$

$\langle S_x \rangle = \chi^{+} \frac{\hbar}{2} S_x \chi =$

$\langle S_y \rangle$

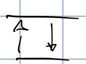
Energy splitting between spin up & down states along z

$-\gamma B_0 \hbar$ electron $= 1.1 \cdot 10^{-4}$ eV $\frac{B_0}{\text{Tesla}}$

$-\gamma B_0 \hbar$ photon $= 6 \cdot 10^{-8}$ eV $\frac{B_0}{\text{Tesla}}$

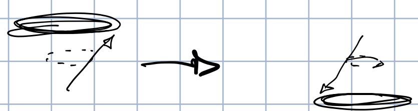
precession frequency $(\omega_0)_{electron} \sim$ 170 GHz

$(\omega_0)_{photon} \sim$ 90 MHz



Spin up
Spin down

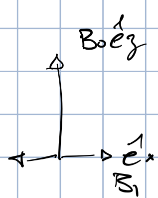
$\begin{pmatrix} a e^{i(\ )t} \\ b e^{-i(\ )t} \end{pmatrix} \longrightarrow |a e^{i(\ )t}| = |a|^2$



paramagnetic resonance

$\vec{B} = B_0 \hat{e}_x + B_1 \hat{e}_x \cos(\omega t)$

tune to be near ω_0



endice probability of switching