Monday:
$$G_{E\ell}(\rho) = \rho^{\ell+1} \sum_{k=0}^{\infty} C_k \rho^k$$

$$\rightarrow \sum \Big[(k+\ell)(k+\ell+1) C_k \rho^{k+\ell-1} \overset{②}{\cancel{- 2(k+\ell+1) C_k \rho^{k+\ell}}}$$

$$\overset{③}{\cancel{+ \frac{\ell^2 K}{E} C_k \rho^{k+\ell}}} \overset{④}{- \ell(\ell+1) C_k \rho^{k+\ell-1}} \Big] = 0$$

gone if $\rho \to 0$

$k=0$: $\ell(\ell+1) C_0 \rho^{\ell-1} - 2(\ell+1)\rho^{\ell} C_0 + \frac{\ell^2 K}{E} C_0 \rho^{\ell} - \ell(\ell+1) C_0 \rho^{\ell-1} = 0$

only $\rho^{\ell-1}$ & they cancel

ρ^{ℓ} terms ② ③ $k=0$

① ④ $k=1$

o/c it equals 0

can drop $k=0$

set $k' = k-1$ in ① & ④

$$\rightarrow \sum_{k'=0}^{\infty} (k'+\ell+1)(k'+\ell+2)\, C_{k'+1} - \ell(\ell+1)\, C_{k'+1} \Big] \rho^{k'+\ell}$$

② & ③

$$\rightarrow \sum C_k \rho^{k+\ell} \left(-2(k+\ell+1) + \frac{e^2 K}{E} \right)$$

call $k' = k$

$$\sum_{k=0}^{\infty} \left[\left((k+\ell+1)(k+\ell+2) - \ell(\ell+1) \right) C_{k+1} + \left(\frac{e^2 K}{E} - 2(k+\ell+1) \right) C_k \right] \rho^{k+\ell}$$

coeff. of $\rho^{k+\ell} = 0$ gives

$$C_{k+1} = \frac{-\frac{e^2 K}{E} + 2(k+\ell+1)}{(k+\ell+1)(k+\ell+2) - \ell(\ell+1)} C_k$$

2 possibilities

① numerator is not 0 for any k, all $C_k \neq 0$ given $C_0 \neq 0$

② numerator vanishes for some k: $-\frac{e^2 K}{E} + 2(k+\ell+1) = 0$

$$C_{k+1} = 0 \to C_{k+2} = C_{k+2} = \dots = 0$$

$$\frac{e^4 k^2}{E^2} = 4(k+l+1)^2$$

$$e^4\left(\frac{-2\mu E}{\hbar^2}\right)\frac{1}{E^2} = 4(k+l+1)^2$$

$$E = \frac{-e^4\, 2\mu}{4\hbar^2(k+l+1)^2} = \frac{-\mu e^4}{2\hbar^2(k+l+1)^2}$$

principle quantum number : $n = k+l+1$

$k = 0, 1, 2, \dots$

$l = 0, 1, 2, \dots$

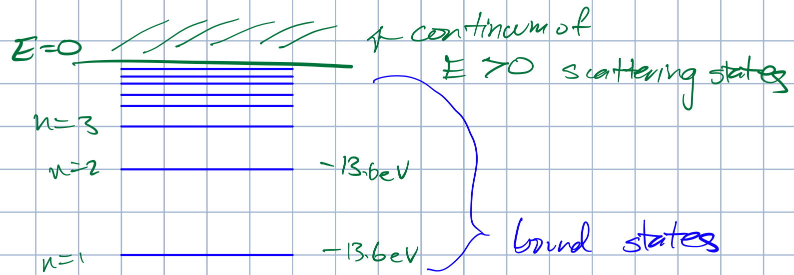
$n = \quad 1, 2, \dots$

$$E_n = \frac{-\mu e^4}{2\hbar^2 n^2} \qquad \text{(cgs units)}$$

$1\,eV = 1.6 \cdot 10^{-12}\,erg = 1.6 \cdot 10^{-19}\,J$

$E_1 \cong -13.6\,eV$

$E_n \cong -\frac{13.6}{n^2}\,eV$



Degeneracy of states H-atom

$n = k+l+1$

$n=1 \qquad k=l=0 \qquad$ unique

$n=2 \qquad k=1,\ l=0$

or $k=0,\ \underline{l=1},$

$m = -1, 0, 1$

$$\phi(\vec{r}) = R_{El}(r)\,Y_{lm}(\theta,\phi)$$

$$m = -l, \dots, l$$

$1+3 = 4$ states @ $n=2$

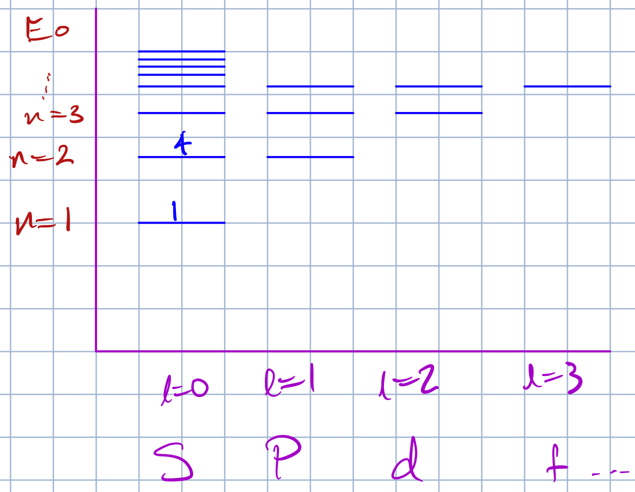
$n=3 \qquad k=2,\ l=0 \to 1$

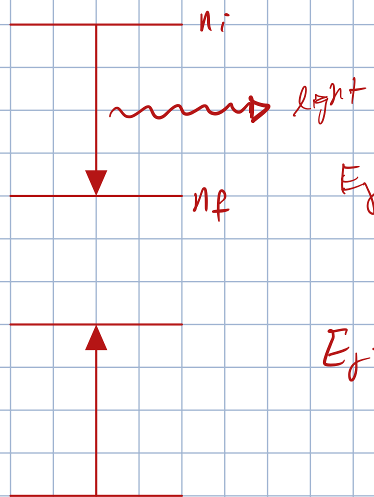
$\qquad k=1,\ l=1 \to 3$

$\qquad k=0,\ l=2 \to \underline{5}$

$\qquad 9$ states

$n=n \qquad n^2$ states w/ energy E_n

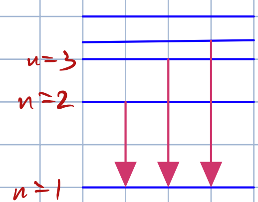




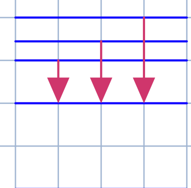
$$E_\gamma = \hbar\omega = E_{n_i} - E_{n_f}$$

$$E_\gamma = \hbar\omega = -13.6\,eV\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$\lambda = \frac{c}{2\pi\omega} \qquad \lambda = \frac{911.8\,\text{Å}}{\frac{1}{n_f^2} - \frac{1}{n_i^2}}$$



Lyman Series
UV



Balmer series – visible

$$|E\rangle \to e^{-iEt/\hbar}|E\rangle$$

$$H_{true} = H_{atom} + H_{radiation} + H_{interactions}$$

our concern

H-atom energy eigenstates