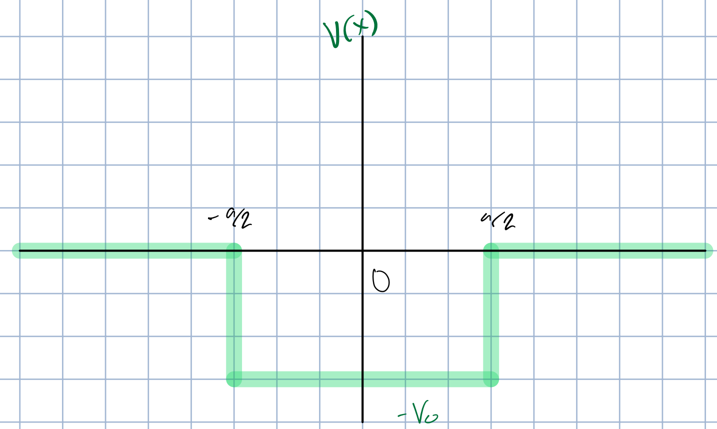


Parity operator

$x \to -x$

$p \to -p$

$[H, P] = 0$

2 sol^{n}s odd & even

even:
$$\phi^{e}(x) = \begin{cases} C_1 e^{\rho x} & x < -a/2 \\ A\cos(qx) & \\ C_1 e^{-\rho x} & x > a/2 \end{cases}$$

odd:
$$\phi^{o}(x) = \begin{cases} C_1 e^{\rho x} & x < -a/2 \\ B\sin(qx) & \\ -C_1 e^{-\rho x} & x > a/2 \end{cases}$$

Match even sol^{n}s

values: $C_1 e^{-\rho a/2} = A\cos(qa/2)$

derivatives: $\rho C_1 e^{-\rho a/2} = qA\sin(qa/2)$ divide by top

$$\rho = q\tan(qa/2)$$

transcendental eq^{n}

$$\rho(E) = \sqrt{\frac{-2mE}{\hbar^2}} \qquad q(E) = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$

do some graphs!

$y = qa$

$\lambda = \frac{2ma^2V_0}{\hbar^2}$ strength of how attractive potential is. width $a \to a$ V_0 strength

$$\lambda - y^2 = \frac{2ma^2V_0}{\hbar^2} - q^2a^2 = -\frac{2mE}{\hbar^2}a^2 = \rho^2 a^2$$

$$\frac{\sqrt{\lambda - y^2}}{y} = \tan(y/2)$$

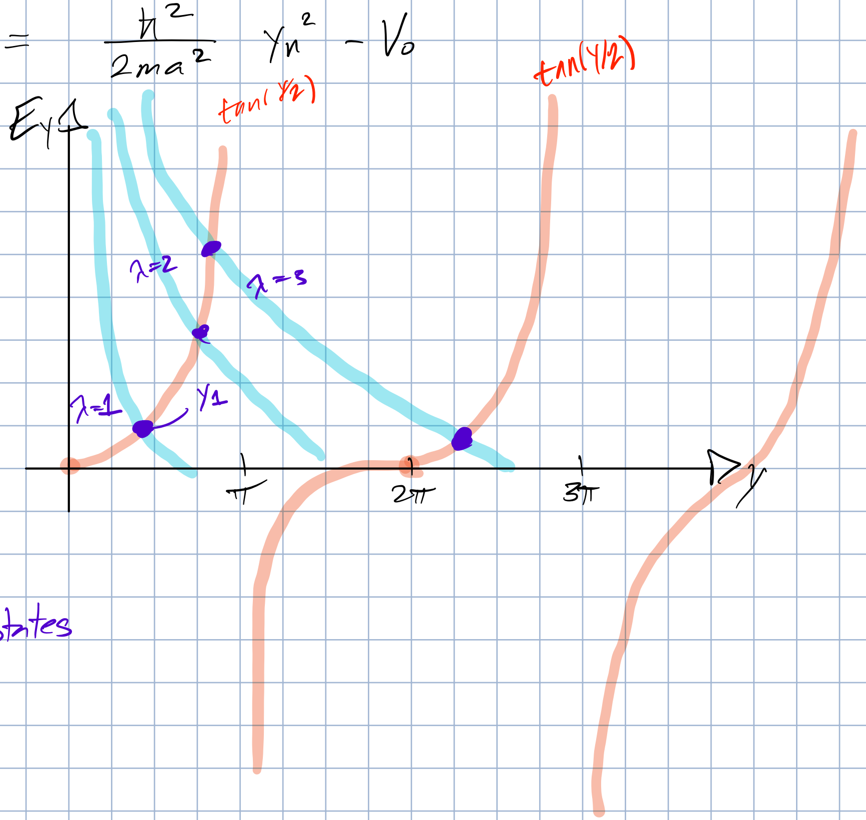
lowest energy: $E = -V_0 \Rightarrow y = 0$

$E \to \infty \qquad y \to \infty$

look @ y sol^{n}s of

can get some E's

$$E_n = \frac{\hbar^2}{2ma^2} y_n^2 - V_0$$

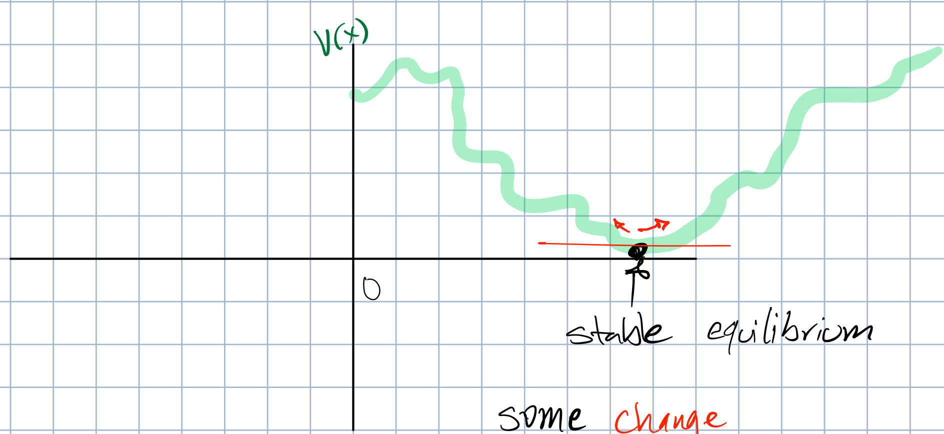


always 1 bound state

can be more, finite states

λ's go w/ potentials

Simple Harmonic Oscillator



stable equilibrium

some change

$$V(x) = \cancel{V(x_{min})}^{\,0} + \cancel{\left.\frac{dV}{dx}\right|_{x_{min}}}^{\,0}(x - x_{min}) + \frac{1}{2}\left.\frac{dV}{dx}\right|_{x_{min}}(x - x_{min})^2 + \dots$$

generally nonzero

$V(x)$

choose x s.t. $x_{min} = 0$

$\rightarrow \hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2$

$\frac{1}{2} m\omega^2 \hat{x}^2 = \frac{1}{2} \left.\frac{dV}{dx}\right|_{x_{min}}$

Solve 2 ways, useful elsewhere

H-atom
angular momentum

Study relation between classical & Quantum mechanics

Solve time indpt. SE

$$-\frac{\hbar^2}{2m} \frac{d^2\phi_E}{dx^2} + \frac{1}{2} m\omega^2 x^2 \phi_E(x) = E\,\phi_E(x)$$

find allowed E for normalizable $\phi_E(x)$

① Simplify eq^n w/ dimensionless variables

② use asymptotic analysis to find behaviors of sol^{n}s @ $x \to \pm\infty$

③ use ② to find series sol^n to differential eq^n

Frobenius method

①

Mass Length Time

$[\omega] = T^{-1}$
$[m] = M$
$[\hbar] = L^2 M T^{-1}$

$\left[\frac{m\omega}{\hbar}\right] = L^{-2}$

can introduce $y = \left(\sqrt{\frac{m\omega}{\hbar}}\right) x$

$$\mathcal{E} = \frac{2E}{\hbar\omega}$$ dimensionless energy

$$\frac{d^2\phi_E}{dy^2} - y^2 \phi_E(y) = -\mathcal{E}\,\phi_E(y)$$

(2) $f(x) \sim g(x)$ as $x \to \infty$ if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1$

exe.

$f(x) = (1+x)e^x$
$g(x) = x e^x$

$\Rightarrow f \sim g$ b/c $\lim_{x\to\infty}\left(\frac{(1+x)e^x}{xe^x}\right) = \lim_{x\to\infty}\left(\frac{(1+x)}{x}\right) = \lim_{x\to\infty}\left(1+\frac{1}{x}\right) = 1$

math - equivalence relation

set $S = \{a, b, c\}$

$\sim$ is equivalence relation if $a \sim a$
$a \sim b$ iff $b \sim a$
$a \sim b,\ b \sim c \implies a \sim c$

example 1

Set $= \mathbb{Z}$ integers

$n \sim m$ if $n - m = 0 \mod 2$

$2 \sim 8$
$-7 \sim 5$

splits into 2

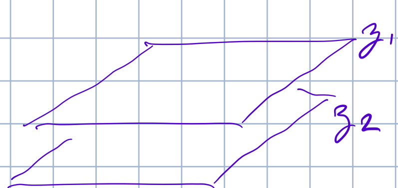
evens $= \{\ldots, -2, 0, 2, 4, \ldots\}$
odds $= \{\ldots, -3, -1, 1, 3, 5, \ldots\}$

these are equivalence relation
" "

example 2

set $= \mathbb{R}^3$

$(x, y, z) \sim (x', y', z')$ if $z = z'$



divide into classes of sol^ns based on behavior @ $x \to \infty$

→ only lives in equivalence classes of f^ns want simple $\Phi_E^{asym}(x) \sim \Phi_E(x)$

$\uparrow$ solves DE

$$\frac{d^2 \Phi_E}{dy^2} - y^2 \phi_E(y) = -\mathcal{E}\, \phi_E(y) \qquad \text{as } y \rightarrow \pm\infty$$

much larger than as $y \rightarrow \pm\infty$

$$\frac{d^2 \phi_E^{asym}}{dy^2} \sim y^2 \phi_E^{asym}(y)$$

only normalizable so that $x \rightarrow \pm\infty$ doesn't blow up

b/c $\int |\Phi_E(x)|^2 = 1$

$$\Phi_E^{asym} = A e^{-y^2/2} \quad \text{for} \quad \frac{d\,\phi_E^{asym}}{dy} = y^2 \phi_E(x)$$

$$\frac{d\,\Phi_E^{asym}}{dy} = -yAe^{-y^2/2}$$

$$\frac{d\,\Phi_E^{asym}}{dy} = y^2 A e^{-y^2/2} - Ae^{-y^2/2} \sim y^2 A e^{-y^2/2}$$