an important point about $V \otimes W$

vector space of all linear combination $v \otimes w$ $\quad v \in V,\ w \in W$

but not all elements of $V \otimes W$ can be written as $v \otimes w$

not all linear combinations are $v \otimes w$

$V = \mathbb{C}^2$

$V \otimes V =$ vector space of all 2×2 complex matrices

choose basis $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \in V \otimes V$ $\quad$ but it is not $v \otimes v'$ for any v, v'

$$v \otimes v' = (v_1 \ \ v_2)\begin{pmatrix} v_1' \\ v_2' \end{pmatrix} = \begin{pmatrix} v_1 v_1' & v_1 v_2' \\ v_2 v_1' & v_2 v_2' \end{pmatrix} \overset{?}{=} \alpha \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$v_1 v_1' = 0$

$\Downarrow$

v_1 or v_1' is zero

$v_1 \cdot v_2' = 1$ or $v_2 v_1' = 1$

but if $v_1 = 0 \to v_1 v_2' = 0$, but it's 1

but if $v_1' = 0 \to v_2 v_1' = 0$, but it's 1!

contradiction!

$V = \mathbb{C}^2$

these kinds of vectors correspond to entangled quantum state

what about linear operators $\mathcal{O}: V \otimes W \to V \otimes W$

$O_V : V \to V$

$O_W : W \to W$

operators take vector in $V \otimes W$

define $(O_V \otimes O_W)\,(v \otimes w) \longrightarrow (O_V v) \otimes (O_W w)$

$V = \mathbb{C}^2$

Two photon states

Recall from E&M: $\quad$ right / left handed circular polarized

$$E_{\substack{RHCP \\ LHCP}} = E_0 \, \mathrm{Re}\left[(\hat{x} + i\hat{y})\, e^{i(kz - \omega t)} \right] \qquad i = e^{i\pi/2}$$

$$= E_0 \left(\hat{x} \cos(kz - \omega t) + \hat{y} \cos(kz - \omega t + \pi/2) \right)$$

QM : these correspond to 1-photon states

$$|R\rangle = \frac{1}{\sqrt{2}}\left(|\hat{x}\rangle + i|\hat{y}\rangle\right)$$
$$|L\rangle = \frac{1}{\sqrt{2}}\left(|\hat{x}\rangle - i|\hat{y}\rangle\right)$$

$$\langle L|L\rangle = \langle R|R\rangle = 1$$
$$\langle L|R\rangle = \langle R|L\rangle = 0$$

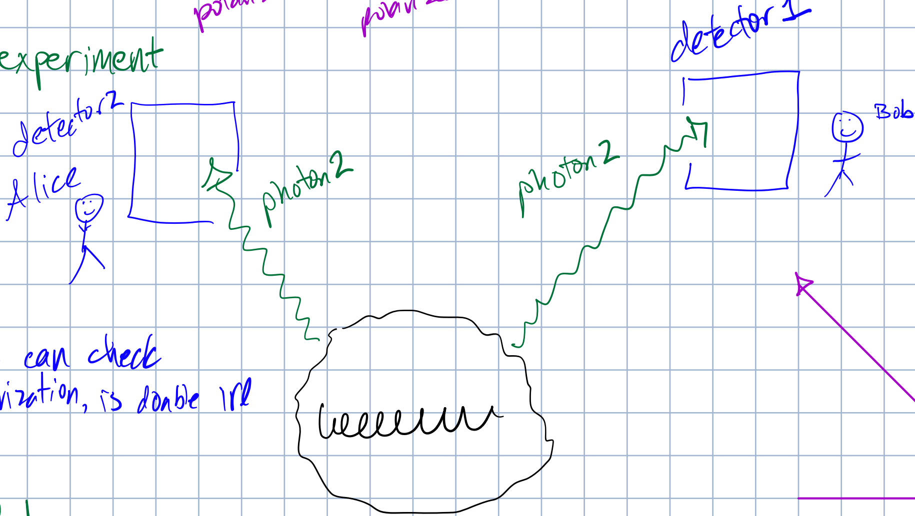
going to look @ state in $\mathbb{C}^2 \otimes \mathbb{C}^2$ (space of 2 photon QM)

1st photon   second photon

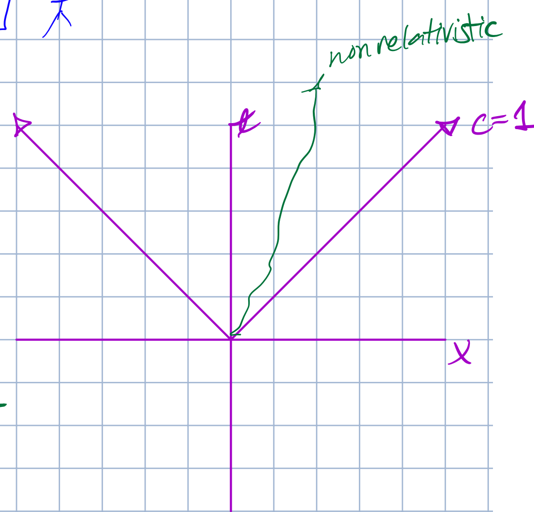
$$|EPR\rangle = \frac{1}{\sqrt{2}}\left(|R\rangle_1 \otimes |L\rangle_2 + |L\rangle_1 \otimes |R\rangle_2 \right)$$

1st right handed polarized   2nd left handed polarized   left   right

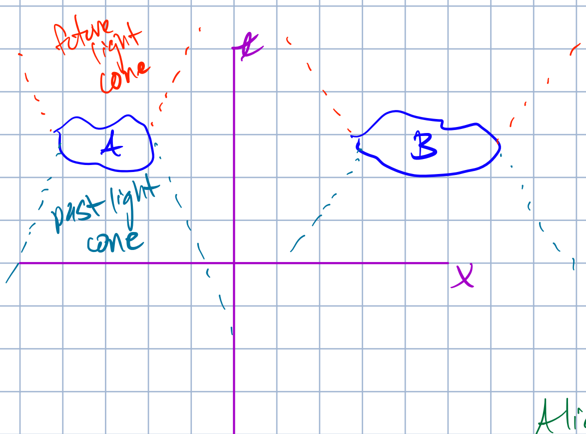
gedanken experiment



detectors can check polarization, is doable irl



want it so that
when Alice measures, has no effect on Bob's measurement
" Bob " Alice



Bob & Alice are spacelike seperated
→ they lie past each other's future light cone
so can't interfere w/ each other

$$\left.\begin{array}{l} t_A > t_B \\ t_B > t_A \end{array}\right\} \text{observer dependent}$$

Alice & Bob can't influence w/each other while taking measurement chose x-pol: $\hat{x} = \begin{pmatrix}1\\0\end{pmatrix}$ +1 eigenvalue

chose y-pol: $\hat{y} = \begin{pmatrix}0\\1\end{pmatrix}$ -1 eigenvalue $\sigma_3 = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$

$\sigma_3|\hat{x}\rangle = +1\,|\hat{x}\rangle$

$\sigma_3|\hat{y}\rangle = -1\,|\hat{y}\rangle$

measuring ±1 eigenvalues shows eigenstates & being polarized along a direction

RHCP $|R\rangle = \frac{1}{\sqrt{2}}(|\hat{x}\rangle + i|\hat{y}\rangle) = \begin{pmatrix}1\\i\end{pmatrix}$ $\sigma_2 = \begin{pmatrix}0&-i\\i&0\end{pmatrix}$

LHCP $|L\rangle = \frac{1}{\sqrt{2}}(|\hat{x}\rangle - i|\hat{y}\rangle) = \begin{pmatrix}1\\-i\end{pmatrix}$

$\sigma_2 - \lambda\mathbb{1} = \begin{vmatrix}-\lambda & -i\\ i & -\lambda\end{vmatrix} = \lambda^2 - 1 \Rightarrow \lambda = \pm 1$

$\lambda = 1$: $\begin{pmatrix}-1 & -i\\ i & -1\end{pmatrix}\begin{pmatrix}v_1\\v_2\end{pmatrix} = \begin{matrix}-v_1 - iv_2 = 0\\ iv_1 - v_2 = 0\end{matrix}$ $-v_1 = iv_2$ if $v_1 = 1, v_2 = i$ $\begin{pmatrix}1\\i\end{pmatrix}$

$\sigma_2|R\rangle = +1|R\rangle$

$\sigma_2|L\rangle = -1|L\rangle$

± 45° linear pol. $|\pm\rangle = \frac{1}{\sqrt{2}}(|\hat{x}\rangle \pm |\hat{y}\rangle)$ $\sigma_1 =$

$\sigma_1|+\rangle = +1\,|+\rangle$

$\sigma_1|-\rangle = -1\,|-\rangle$

Alice & Bob can't influence

→ assume Alice & Bob can measure $\sigma_i \otimes \sigma_j$ for any i, j

$\underset{\text{Bob}}{\sigma_2} \otimes \underset{\text{Alice}}{\sigma_3}$ measures R, L for photon 1; $\hat{x}, \hat{y}$ for photon 2

Measure R, L (or $\sigma_2 \otimes \sigma_2$) → 4 possibilities

Bob	Alice	prob
R	R	0
R	L	½
L	R	½
L	L	0

$$\text{prob}(R,L) = \left|\left({}_1\langle R| \otimes {}_2\langle L|\right)|EPR\rangle\right|^2$$

$$= \left|\left({}_1\langle R| \otimes {}_2\langle L|\right) \otimes \frac{1}{\sqrt{2}}\left(|R\rangle_1 \otimes |L\rangle_2 + |L\rangle_1 \otimes |R\rangle_2\right)\right|^2$$

$$= \left|\left({}_1\langle R| \otimes {}_2\langle L|\right) \otimes \frac{1}{\sqrt{2}}\left(|R\rangle_1 \otimes |L\rangle_2\right) + \left({}_1\langle R| \otimes {}_2\langle L|\right) \otimes \frac{1}{\sqrt{2}}\left(|L\rangle_1 \otimes |R\rangle_2\right)\right|^2$$

$$= \left|\frac{1}{\sqrt{2}}\left({}_1\langle R| \otimes {}_2\langle L|\right) \otimes \left(|R\rangle_1 \otimes |L\rangle_2\right) + \frac{1}{\sqrt{2}}\left({}_1\langle R| \otimes {}_2\langle L|\right)\left(|L\rangle_1 \otimes |R\rangle_2\right)\right|^2$$

$$= \left|\frac{1}{\sqrt{2}}\left({}_1\langle R|R\rangle_1 \otimes {}_2\langle L|L\rangle_2\right) + \frac{1}{\sqrt{2}}\left({}_1\langle R|L\rangle_1 \otimes {}_2\langle L|R\rangle_2\right)\right|^2$$

$$= \left|\frac{1}{\sqrt{2}}(1\cdot 1) + \frac{1}{\sqrt{2}}(0\cdot 0)\right|^2 = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

$$= \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

Alice & Bob each see a random distribution: $\frac{1}{2}R$ $\frac{1}{2}L$

if Bob gets R, Alice gets L

despite being space indpt.

Einstein: spukhafte Fernwerkung

spooky long dist. effect

EPR → entangled states