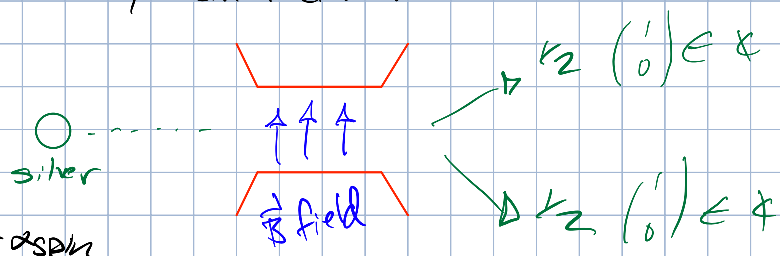
TA → go

Two state systems

① photon polarization

② spin degree of freedom of electrons
spin: "quantum angular momentum"
$

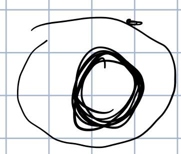
in practice: silver atoms by Stern & Gerlach



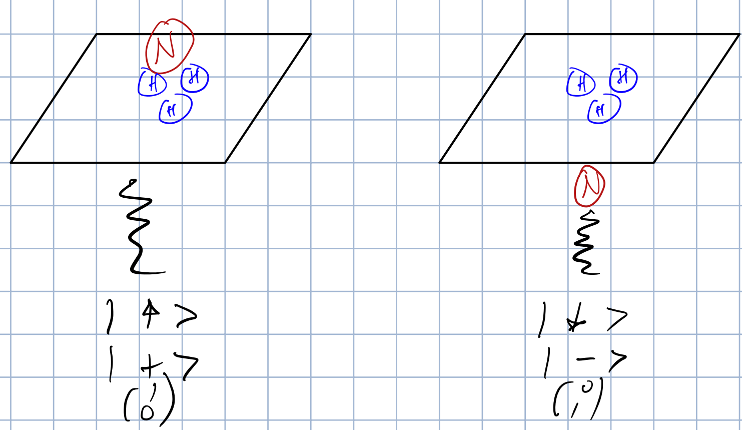
electron – magnetic moment ∝ spin

silver – 47 electrons

46+1



③ Ammonia molecule (NH_3) (Feynman vol III)



$\mathcal{H} = \mathbb{C}^2$

Hamiltonian H – 2×2 Hermitian matrix
Phys. observable $\mathcal{O} \sim 2\times2$ Hermitian matrix

Time evolution

Hamiltonian H - Herm. 2x2 matrix

there is a basis s.t. $H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$ $E_1, E_2 \in \mathbb{R}$

$\hookrightarrow$ correspond to measurable energies

$H | E_i \rangle = E_i | E_i \rangle$

$\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = E_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|E_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in \mathbb{C}^2$
$|E_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \in \mathbb{C}^2$

$\hookrightarrow$ energy eigenstates

at time $t=0$, $|\psi(0)\rangle = |E_1\rangle$

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H |\psi(t)\rangle$$

$$|\psi(t)\rangle = e^{-iE_1 t/\hbar} |E_1\rangle$$

is a sol^n $\rightarrow$ same as E_1, affects no physics

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = i\hbar \cdot \left(\frac{-iE_1}{\hbar}\right) \cdot |\psi(t)\rangle = H |\psi(t)\rangle$$

using $E_1 |\psi(t)\rangle = H |\psi(t)\rangle$

$\rightarrow$ just a phase shift

complex # w/ norm 1 $\rightarrow$ phase

$p = |\langle \lambda_i | \psi(t) \rangle|^2$ is indpt. of phase of $|\psi(t)\rangle$

so, in QM,

state $|\psi(t)\rangle \in \mathcal{H}$, but physics corresponds to ray in $\mathcal{H}$

set of all vectors $e^{i\phi} |\psi(t)\rangle$

multiply by $e^{i\phi}$ $\rightarrow$ "up to a phase"

$$p = |\langle \lambda | e^{i\phi} | \psi(t) \rangle|^2 = \underbrace{e^{i\phi} e^{-i\phi}}_{1} |\langle \lambda | \psi(t) \rangle|^2$$

the $|\ |^2$ = multiply by complex conjugate.

any phase ϕ will have no physical meaning; physical meaning only from probability.

$e^{i\phi}$ is irregardless of time dependence different vector in Hilbert, represent same thing

at time $= 0$ $\to$ $|\psi(0)\rangle = \alpha|E_1\rangle + \beta|E_2\rangle$ $\alpha^2+\beta^2=1$ $\alpha,\beta \in \mathbb{R}$

$\hookrightarrow$ no definite energy. $|E_1\rangle$ sometimes, $|E_2\rangle$ sometimes $|\alpha|^2+|\beta|^2=1$

$$|\psi(t)\rangle = c_1(t)\cdot|E_1\rangle + c_2(t)\cdot|E_2\rangle$$

w/c $|E_1\rangle$ & $|E_2\rangle$ are basis b/c they're eigenstates

boundary: $c_1(0)=\alpha$, $c_2(0)=\beta$

Schro. Eq. $\to$ $i\hbar \frac{\partial}{\partial t}\left(c_1(t)\cdot|E_1\rangle + c_2(t)\cdot|E_2\rangle\right) = H\left(c_1(t)\cdot|E_1\rangle + c_2(t)\cdot|E_2\rangle\right)$

$$i\hbar\frac{dc_1}{dt}|E_1\rangle + i\hbar\frac{dc_2}{dt}|E_2\rangle = c_1(t)\,\overbrace{E_1|E_1\rangle}^{H|E_1\rangle} + c_2(t)\,E_2|E_2\rangle$$

$$\begin{pmatrix}E_1 & 0\\0 & E_2\end{pmatrix}\begin{pmatrix}E_1\\0\end{pmatrix} = \begin{pmatrix}E_1\\0\end{pmatrix}$$

$$\begin{pmatrix}E_1 & 0\\0 & E_2\end{pmatrix}\begin{pmatrix}0\\E_2\end{pmatrix} = \begin{pmatrix}0\\E_2\end{pmatrix}$$

$\langle E_1|$ both sides

$$i\hbar\frac{dc_1}{dt} = c_1(t)E_1$$

same w/ $\langle E_2|$

$$i\hbar\frac{dc_2}{dt} = c_2(t)E_2$$

b/c $|E_1\rangle$ & $|E_2\rangle$ are O-N

$$\longrightarrow c_1(t) = c_1(0)\,e^{-iE_1t/\hbar} = \alpha e^{-iE_1t/\hbar}$$
$$c_2(t) = \beta e^{-iE_2t/\hbar}$$

$$|\psi(t)\rangle = \alpha e^{-iE_1t/\hbar}|E_1\rangle + \beta e^{-iE_2t/\hbar}|E_2\rangle$$

can now describe state $|\psi\rangle$ as fx of time $|\psi(t)\rangle$

$H = \begin{pmatrix}E_1 & 0\\0 & E_2\end{pmatrix}$ $\Longrightarrow$ if $E_1 \neq E_2$ $\longrightarrow$ phase of both are different, can turn it into physical effect

$\vec{B} = B_0\hat{e}_z$

different energies

measure spin along $\hat{e}_x$?

let's measure $\sigma = \begin{pmatrix}0&1\\1&0\end{pmatrix}$

prob. of measuring eigenvalue 1 of σ

take eigenvector of σ w/ eigenvalue 1

$$p = \left|\underbrace{\tfrac{1}{\sqrt2}\left(\langle E_1| + \langle E_2|\right)}_{\frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix}}|\psi(t)\rangle\right|^2$$

$\sigma\cdot\frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix}$

$$= \frac{1}{2}\left|\alpha e^{-iE_1t/\hbar} + \beta e^{-iE_2t/\hbar}\right|^2$$

$\langle E_1|\psi(t)\rangle = \alpha e^{-iE_1\frac{t}{\hbar}}$

$\langle E_2|\psi(t)\rangle = \beta e^{-iE_2\frac{t}{\hbar}}$

$$= \frac{1}{2}\left(\alpha^2 + \beta^2 + \alpha\beta\, 2\cos\left(\frac{(E_2 - E_1)t}{\hbar}\right)\right)$$

time dependent probability

$$|A+B|^2 = |A|^2 + |B|^2 + |AB|^2 + |BA|^2$$

$|\alpha e^{itE_1/\hbar}|^2 \rightarrow \alpha^2$ b/c phase goes away

$|\beta e^{itE_2/\hbar}|^2 \rightarrow \beta^2$

$$\alpha\beta \cdot \underbrace{\underbrace{e^{iE_1 t/\hbar}}_{A^*} \cdot \underbrace{e^{-iE_2 t/\hbar}}_{B}}_{\text{complex conj of first}} + \underbrace{\alpha\beta\, \underbrace{e^{-iE_1 t/\hbar}}_{A} \cdot \underbrace{e^{iE_2 t/\hbar}}_{B^*}}_{\text{complex conj of second}}$$

$$\alpha\beta\left(e^{\frac{(E_1-E_2)ti}{\hbar}} + e^{\frac{(E_2-E_1)ti}{\hbar}}\right) = \alpha\beta\left(\cos\left(\frac{(E_1-E_2)t}{\hbar}\right) + i\sin\left(\frac{(E_1-E_2)t}{\hbar}\right)\right.$$

$$\left. + \cos\left(\frac{(E_2-E_1)t}{\hbar}\right) + i\sin\left(\frac{(E_2-E_1)t}{\hbar}\right)\right)$$

$$\cos(A-B) + i\sin(A-B) + \cos(B-A) + i\sin(B-A)$$

$\cos(-\theta) = \cos\theta$ $\qquad$ $\sin(-\theta) = -\sin\theta$

$$\cos(B-A) - i\sin(B-A) + \cos(B-A) + i\sin(B-A)$$

$$2\cos(B-A)$$

$$= 2\cos\left(\frac{(E_2-E_1)t}{\hbar}\right)$$

prob oscillates w/ freq. $\Rightarrow$ $\omega = \frac{E_2 - E_1}{\hbar}$

NH_3

$E_2 - E_1 \sim 10^{-4}\ eV$

$\lambda \sim 1.3\ cm$

energy eigenstate is an eigenstate of the Hamiltonian H

describes time evolution