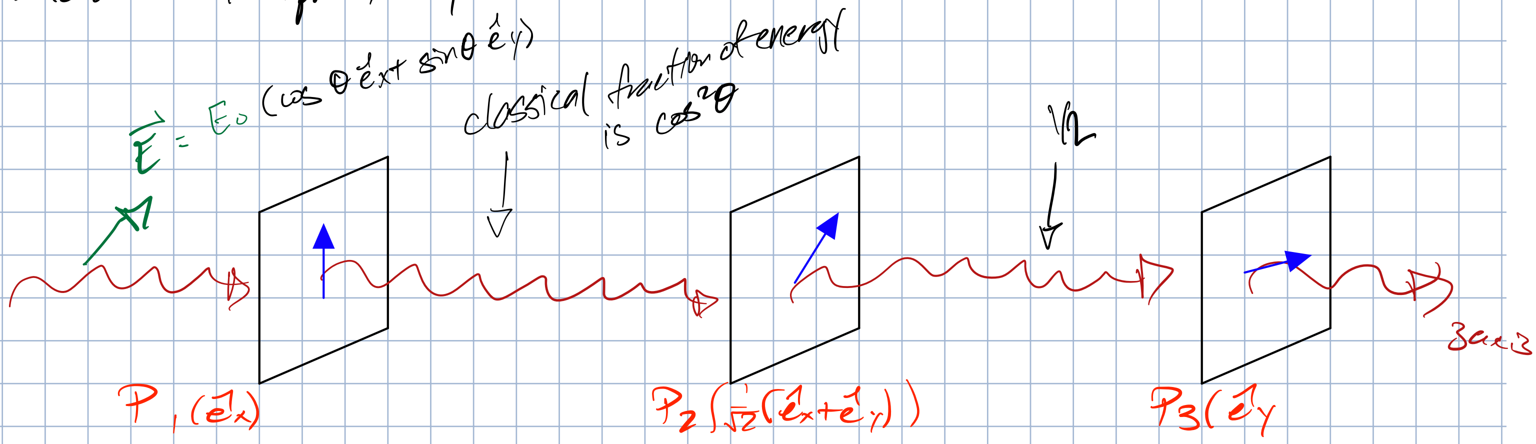
psets due 11:59pm Friday



1st Postulate: state of system ~ $|\psi\rangle$ in Hilbert Space

$\mathcal{H} \in \mathbb{C}^2$

2nd Postulate: Physical observable ~ Hermitian op. $\underline{O}$ measurement yields eigenvalue λ_i of O w/ prob. $p_i = |\langle\lambda_i|\psi\rangle|^2$

w/ $\langle\psi|\psi\rangle = 1$

in this system $P_1(\vec{e}_x) \sim O_x = \begin{pmatrix}1&0\\0&0\end{pmatrix}$ in basis of $\mathbb{C}^2$ $\hat{x} \sim \begin{pmatrix}1\\0\end{pmatrix}$ $\hat{y} \sim \begin{pmatrix}0\\1\end{pmatrix}$

$\rightarrow O_x$ = proj. operator onto $|\hat{x}\rangle$ $\qquad O_x|\hat{x}\rangle = |\hat{x}\rangle$

$|\theta\rangle = \cos\theta|\hat{x}\rangle + \sin\theta|\hat{y}\rangle$

prob. of being polarized along $|\hat{x}\rangle$ (passing thru 1st polarizer)

$$p = |\langle\hat{x}|\theta\rangle|^2 = |\langle\hat{x}|\cos\theta|\hat{x}\rangle + \sin\theta|\hat{y}\rangle|^2 = \cos^2\theta$$

3rd Postulate: If a measurement of O gives eigenvalue λ immediately after measurement, system is in state $|\lambda_i\rangle$

P_2 ~ Hermitian operator

proj. operator onto state of being polarized along 45° or $\underbrace{\frac{1}{\sqrt{2}}(|\hat{x}\rangle + |\hat{y}\rangle)}_{|\psi_+\rangle}$

$$O_+ = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

$|\psi_-\rangle$

any vector in $\mathbb{C}^2$: $\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{(\alpha+\beta)}{\sqrt{2}} \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} + \frac{(\alpha-\beta)}{\sqrt{2}} \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$

projection along $|\psi_+\rangle$ of α is $\frac{\alpha+\beta}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = O_+ \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

$O_+ |\psi_+\rangle = \mathbb{1}$

$$\begin{pmatrix} a\alpha + b\beta \\ c\alpha + d\beta \end{pmatrix} = \begin{pmatrix} \alpha+\beta/2 \\ \alpha+\beta/2 \end{pmatrix} \rightarrow \begin{matrix} 2a\alpha + 2b\beta = \alpha+\beta \\ 2c\alpha + 2d\beta = \alpha+\beta \end{matrix}$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \begin{pmatrix} \alpha & \beta \end{pmatrix}$$

prob. of going thru P_2

$$P = |\langle \psi_+ | \hat{x} \rangle|^2$$
$$= \left| \tfrac{1}{\sqrt{2}} (\langle \hat{x}| + \langle \hat{y}|) | \hat{x} \rangle \right|^2$$
$$= \left| \tfrac{1}{\sqrt{2}} \right|^2 = \tfrac{1}{2}$$

state is now $|\psi_+\rangle = \text{Normalize}(O_+ O_x |\theta\rangle)$

fraction of energy going thru P_1 & P_2 is $\frac{1}{2}\cos^2\theta$

photons hard! Bye!

4^{th} Postulate: time evolution of state $|\psi(t)\rangle$ is determined by Schrödiger eq$^{\underline{n}}$

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H |\psi(t)\rangle$$

$\hookrightarrow$ Hamiltonian - Hermitian operator whose eigenvalues are allowed energies

Linear Algebra Overview

1. def$^{\underline{n}}$ of a group & its role defining vector spaces
2. dual vectors & linear operators on vector spaces → Why Dirac?
3. change of basis
4. tensor product $V \otimes W$ → central to EPR paradox
5. $\mathbb{C}^N$ → Hilbert Space $L^2(\mathbb{R})$

$N \to \infty$