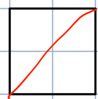
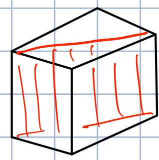
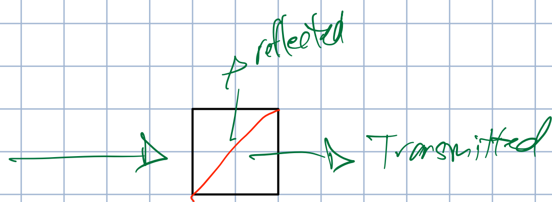
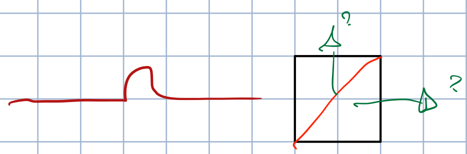
light beams, beam splitters, polarizers + Einstein & photons

Beam splitter









maybe there is some preference?

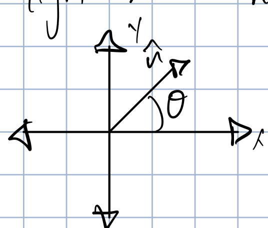
Transmission or Reflection is probabilistic

photon	
1	T
2	T
3	R
4	T
5	R
6	R
⋮	⋮

} # photons large, $\frac{1}{2}$ R, $\frac{1}{2}$ T

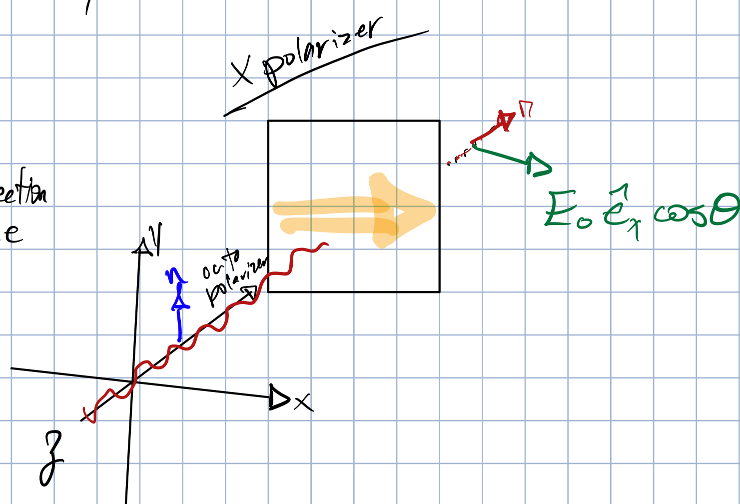
light along z-axis $\qquad \vec{E} = \overset{\text{magnitude}}{E_0}\ \underset{\text{polarization, } \perp \text{ to motion}}{\hat{n}}\ \cos(kz - \omega t)$

light w/ $\quad \hat{n} = \cos\theta\, \hat{e}_x + \sin\theta\, \hat{e}_y$

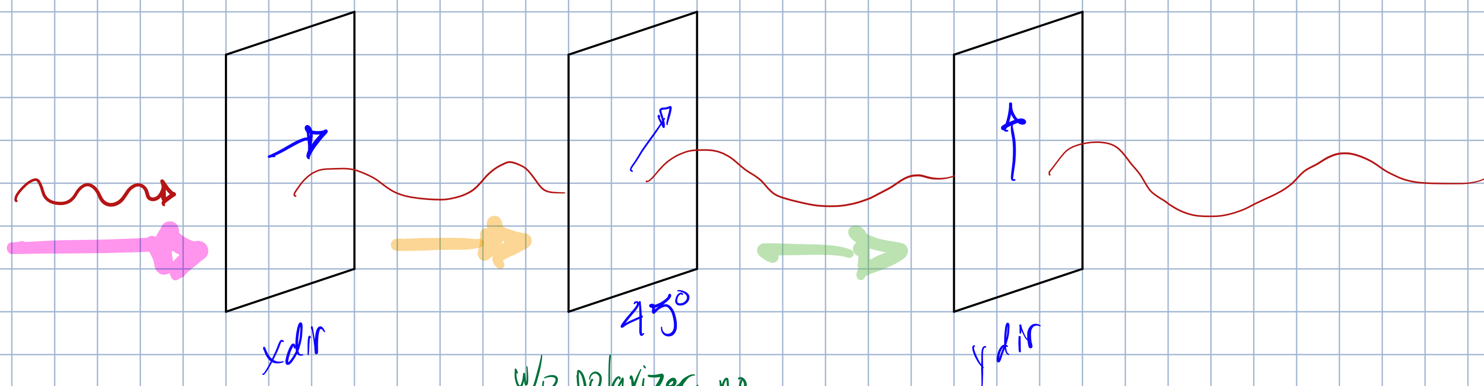


linear polarizer

preffered direction only along line



Series of polarizers



w/o polarizer, no light passes thru

w/ polarizer light @ 45°

2 polarizers

$P(\hat{n})$ linear polarizer along $\hat{n}$

1^{st} : $P_1(\hat{e}_x)$

2^{nd} : $P_2\left(\frac{1}{\sqrt{2}}(\hat{e}_x+\hat{e}_y)\right)$

$\to \cos\theta\,\hat{e}_x + \sin\theta\,\hat{e}_y$

P_1: $E_0\,\hat{n} \longrightarrow E_0\,\hat{e}_x\cos\theta$

energy $\propto E_0^2\cos^2\theta$

original energy: E_0^2

lost b/c photons leave or heat up polarizer

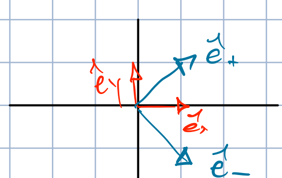
P_2: $E_0\hat{e}_x\cos\theta \longrightarrow E_0\cdot\frac{1}{\sqrt{2}}\cos\theta\;\hat{e}_+$

$P_2\left(\frac{1}{\sqrt{2}}(\hat{e}_x+\hat{e}_y)\right) = P(\hat{e}_+)$

$\hat{e}_x = \frac{1}{\sqrt{2}}(\hat{e}_+ + \hat{e}_-)$

$\hat{e}_+ = \frac{1}{\sqrt{2}}(\hat{e}_x+\hat{e}_y)$

$\hat{e}_- = \frac{1}{\sqrt{2}}(\hat{e}_x-\hat{e}_y)$



only thing being transmitted is in $\hat{e}_+$ direction. $\hat{e}_+$ is $\perp$ to $\hat{e}_-$

light along $\hat{e}_-$ is absorbed or reflected now energy $\propto \frac{E_0^2 \cos^2\theta}{2}$

this is all for classical mechanics.

How does QM make sense of this?

1^{st} postulate of QM:

The state of a physical system is described by a vector $|\psi\rangle$ in a Hilbert Space $\mathcal{H}$

For our system - 2 polarization states of a photon.

$\mathcal{H} = \mathbb{C}^2$ — 2D complex vector space

$|\psi\rangle \in \mathbb{C}^2$
'ket'

concretely as $\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$ $\alpha, \beta \in \mathbb{C}$

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \alpha \cdot \begin{pmatrix}1\\ 0\end{pmatrix} + \beta \begin{pmatrix}0\\ 1\end{pmatrix}$$

orthonormal basis

Natural inner product generalizing $\vec{v}\cdot\vec{w}$ of $\vec{v}, \vec{w} \in \mathbb{R}^3$

$\vec{v} = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$ $\vec{w} = \begin{pmatrix}\gamma\\ \delta\end{pmatrix}$

always true in QM

inner product: $\langle w | v \rangle = w^{+} v = (\gamma^* \ \ \delta^*) \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$

$$= \gamma^* \cdot \alpha + \delta^* \beta$$

$|v\rangle$ bra — vector, state describe what's going on
$\langle w|$ ket — 'dual vector'

$\langle w | v \rangle$ bra (c) ket

$$\vec{E} = E_0 \hat{n} = E_0 (\cos\theta\, \hat{e}_x + \sin\theta\, \hat{e}_y)$$

in 3-D

$\Longrightarrow$ photon is state in $\mathbb{C}^2$

in 2D complex vector space

$$|\theta\rangle = \cos\theta\, |\hat{x}\rangle + \sin\theta\, |\hat{y}\rangle = \begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$$

can choose $|\hat{x}\rangle = \begin{pmatrix}1\\ 0\end{pmatrix}$ $|\hat{y}\rangle = \begin{pmatrix}0\\ 1\end{pmatrix}$

2^{nd} Postulate of QM:

???

Physical observables correspond to Hermitian operator σ on $\mathcal{H}$

(Hermitian matrix in $\mathbb{C}^2$)

a measurement of σ in a state $|\psi\rangle$ yields one of the eigen values λ_i of σ $\left(\sigma |\lambda_i\rangle = \lambda_i |\lambda_i\rangle \right)$

with probability P_i

$$P_i = | \langle \lambda_i | \psi \rangle |^2$$ w/ $|\psi\rangle$ is normalized: $\langle \psi | \psi \rangle = 1$