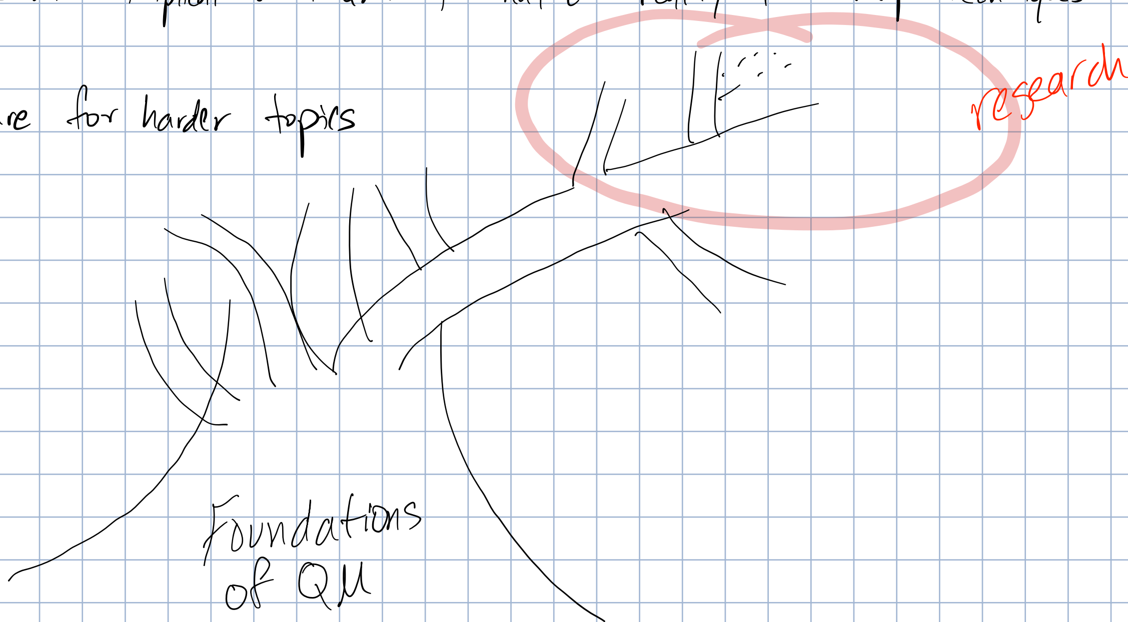
Objectives

Learn how to use ideas & techniques of QM to describe physical phenomena

Learn how to apply variety of math techniques in (linear algebra, Fourier ...)

Appreciate implications of QM for nature of reality & develop techniques used in QIM

Prepare for harder topics



photon pol. → 2 state systems → EPR → Bell's thm →

SE → 1-D potentials → 3d potentials & angular momentum → spectrum of hydrogen

MRI ← Spin, B field → atomic spectrum →

→ Scattering in 1D, tunneling → 3-D analogs → nuclear decay

→ electron microscope

$\hat{H}\,\Psi(x,t) = i\hbar \frac{\partial}{\partial t}\left(\Psi(x,t)\right)$ $\Psi(x,t) = \phi(x)\, f(t)$

linear eq^n $\Rightarrow$ if Ψ_1 & Ψ_2 are sol^{n}s, so is $\Psi_1 + \Psi_2$ a sol^n

$\Rightarrow$ like adding vectors in vector space

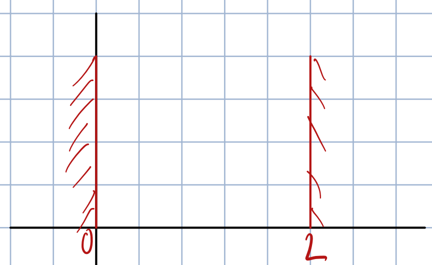
if Ψ is a sol^n, so is $\alpha\Psi$ $\alpha \in \mathbb{C}$

we got 2 eq^n's

1. $i\hbar \frac{df}{dt} = E f(t) \implies f(t) = e^{-iEt/\hbar}$

2. $\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + U(x)\right)\phi_E(x) = E\,\phi_E(x)$

"particle in a box" $V(x) = \begin{cases} \infty & x<0,\ x\geq L \\ 0 & 0<x<L \end{cases}$



let's look @ $0 \leq x \leq L$ $\phi(0) = \phi(L)$

vanishes outside box $\phi = 0$ @ $V = \infty$

b/c $V(x) = 0$, our eq^n: $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\phi_E(x) = E\,\phi_E(x)$ $k^2 = \frac{2mE}{\hbar^2}$

$\frac{d^2}{dx^2}\phi_E(x) = -k^2\,\phi_E(x)$

ODE $\Rightarrow$ $\phi(x) = A\cos(kx) + B\sin(kx)$

$\phi(0) = A = 0$

$\phi(L) = B\sin(kL) = 0$

if $B = 0 \Rightarrow \Psi(x) = 0$, $\Psi = 0$ ew, not fun

if $\sin(kL) = 0$, we fix the allowed values of $k \Rightarrow E$

$\Rightarrow kL = n\pi \Rightarrow k = \frac{n\pi}{L}$ $n = 1, 2, 3, \ldots$

no 0 b/c no fun

no (-) b/c just changes sign, not significant

we have found SE eq^{n}'s

$$\phi_n(x) = B\sin\left(\frac{n\pi x}{L}\right) \quad \& \quad f(t) = e^{\frac{-iE_n t}{\hbar}}$$

for n=1, 2, ...

$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2mL}$$

$$\int_0^L |\psi(x,t)|^2\,dx = 1$$

want prob. to be 1

$f_n(t)$ drops out

$$= \int_0^L B^2 \sin^2\left(\frac{n\pi x}{L}\right)dx = 1$$

$$\xrightarrow{\text{math}} B = \sqrt{\frac{2}{L}}$$

now have normalized sol^{n}'s w/ $B = \sqrt{\frac{2}{L}}$

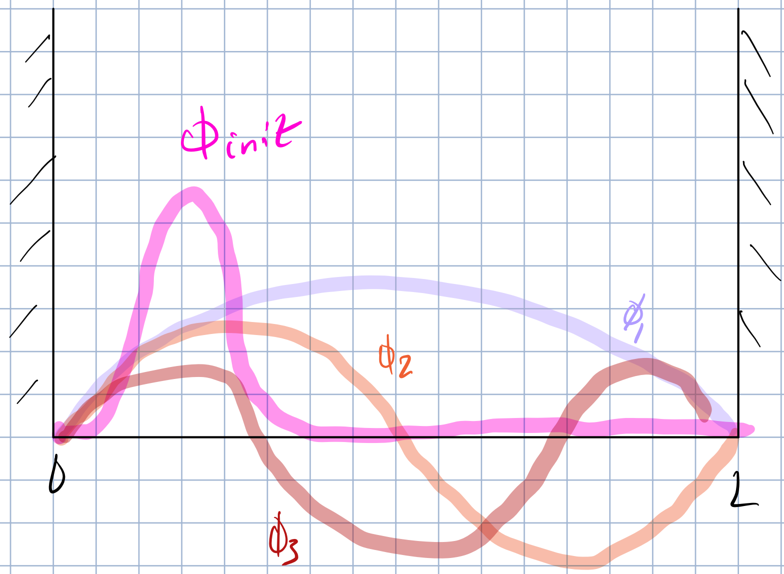
$$\phi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{2}\right) \quad \& \quad f_n(t) = e^{\frac{-iE_n t}{\hbar}}$$

general sol$^{\underline{n}}$ of time dependent SE $\Longrightarrow \sum_{n=1}^{\infty} c_n \phi_n(x) f_n(t)$

Wolfram alpha $\rightarrow$ ok for integrals, no linear algebras

$$\hat{H}\phi = E\phi$$

sol$^{\underline{n}}$ to time dependent eq$^{\underline{n}}$



$$E_n = n^2 \left(\frac{\hbar^2 \pi^2}{2mL^2}\right)$$

energy increases exponentially

$$P(x,t) = |\psi_n(x,t)|^2 = |\phi_n(x)|^2$$

stationary state

for fixed n ind. time

$$\phi_{init}(x) = \sum_n c_n \phi_n(x) = \sum_n c_n \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{2}\right)$$

$$\psi_{init}(x,0) = \phi_{init}(x) \longrightarrow \psi(x,t) = \sum_n c_n \phi_n(x) \, e^{-\frac{iE_n t}{\hbar}}$$

$$\psi(x,0) = \sum_n c_n \phi_n(x)$$

$$\psi(x,t) = \sum_n c_n \phi_n(x) \cdot e^{-\frac{iE_n t}{\hbar}}$$