(1)

$[A, B] = 0 \rightarrow$ find basis in which both A & B are diagonal

$[A, B] \neq 0 \rightarrow$ can't do this

$[\sigma_2, \sigma_3] \neq 0$

σ_3 eigenvalues $\begin{pmatrix}1\\0\end{pmatrix} \begin{pmatrix}0\\1\end{pmatrix}$

σ_2 eigenvectors $\frac{1}{\sqrt{2}}\begin{pmatrix}i\\1\end{pmatrix}$ $\frac{1}{\sqrt{2}}\begin{pmatrix}-i\\1\end{pmatrix}$

once measuring σ_3, state can be described as linear combination of σ_2 $\rightarrow$ not definite

(2) Alice measures σ_3. She is measuring eigenvalues of $\sigma_3 \otimes \mathbb{1}$ on $V_A \otimes V_B$

Vector space of Alice

" " Bob

eigenstates of $\sigma_3 \otimes \mathbb{1}$

$\rightarrow$ $+1$ is $|\hat{x}\rangle_A \otimes |\hat{x}\rangle_B$

& $|\hat{x}\rangle_A \otimes |\hat{y}\rangle_B$

2 eigenstates w/ same eigenvalue $+1$

Flip coins 3 times prob 1H, 2T is 3/8

THH
HTH
HHT

$\}$ 3 ways but 8 ways of flipping 3 coins

$$|\text{ent}\rangle = \overset{(x+iy)}{|R\rangle_A} \otimes \overset{(x+iy)}{|R\rangle_B}$$

$$= \tfrac{1}{2}\left(|\hat{x}\rangle_A \otimes |\hat{x}\rangle_B + i\,|\hat{x}\rangle_A \otimes |\hat{y}\rangle_B + i\,|\hat{y}\rangle_A \otimes |\hat{x}\rangle_B - |\hat{y}\rangle_A \otimes |\hat{y}\rangle_B \right)$$

operator $\sigma_3 \otimes \mathbb{1}$

what is probability Alice measures $+1$?

$+1$ eigenstates are $|\hat{x}\rangle_A \otimes |\hat{x}\rangle_B$ & $|\hat{x}\rangle_A \otimes |\hat{y}\rangle_B$

First prob. $|\hat{x}\rangle_A \otimes |\hat{y}\rangle_B$

$$= |(\langle\hat{x}|_A \otimes \langle\hat{y}|_B)|unent\rangle|^2$$

$$= |\tfrac{1}{2}|^2 = \tfrac{1}{4}$$

Second prob. $|\hat{x}\rangle_A \otimes |\hat{y}\rangle_B$

$$= |(\langle\hat{x}|_A \otimes \langle\hat{y}|_B)|enent\rangle|^2 = |\tfrac{i}{2}|^2 = \tfrac{1}{4}$$

total $\tfrac{1}{4} + \tfrac{1}{4} = \tfrac{1}{2}$

$$({}_v\langle v'| \otimes {}_w\langle w'|)(|v\rangle_v \otimes |w\rangle_w)$$
$$= \langle v'|v\rangle_v \cdot \langle w'|w\rangle_w$$

Dawg, this is just math. Newton? Maxwell?

A particle of mass m moving in 1-D in potential $U(x)$

Classical

$$H(p,x) = \frac{p^2}{2m} + V(x)$$
$$\frac{dx}{dt} = \frac{\partial H}{\partial p} = p/m \qquad p = mv$$
$$\frac{dp}{dt} = -\frac{\partial H}{\partial x} = -\frac{\partial V}{\partial x} = F = ma = \qquad a = \frac{1}{m}\cdot\frac{dp}{dt}$$

Quantum, put hats

$$\hat{H}(\hat{p},\hat{x}) = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

$\hat{H}, \hat{p}, \hat{x}$ are operators acting on a complex wave fn $\Psi(x,t)$ obeying

$$\boxed{\hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t}} \qquad \hat{H} \text{ is an operator}$$

$$\hat{x}\,\Psi(x,t) \overset{\text{no hat}}{=} x\,\Psi(x,t)$$

$$\hat{p}\,\Psi(x,t) = -i\hbar\frac{\partial}{\partial x}[\Psi(x,t)] \longrightarrow ??? \text{ porque?}$$

p is conserved quantity when there is translation invariance

$$\hat{H} = \frac{\hat{p}^2}{2m} + U(\hat{x})$$
$$= \left(-\frac{\hbar^2}{2m}\frac{\partial}{\partial x^2} + U(x)\right)$$

$$\Rightarrow \hat{H}\,\psi(x,t) = \left(-\frac{\hbar^2}{2m}\frac{\partial}{\partial x^2} + U(x)\right)\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

$\Rightarrow$ time dependent Schrödinger eqn

physical interpretation $\Rightarrow$ $|\psi(x,t)|^2 dx =$ prob. of finding particle between x & $x+dx$

prob density
$$P(x,t) = |\psi(x,t)|^2$$

demand $$\int_{-\infty}^{\infty} P(x,t)\,dx = \int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1$$

if $\int_{-\infty}^{\infty} |f(x)|^2$ is finite, we write $\int_{-\infty}^{\infty} |f(x)|^2 dx < \infty$

are called square integrable, $f \in L^2(\mathbb{R})$

norm is finite
Labegue?

this is a Hilbert space
∞-dimensional

$$\int_{-\infty}^{\infty} |f(x)|^p dx$$

now going to be solving DE's

Solve/Simplify time dependent SE by technique (sep. of vars)

$$\Psi(x,t) = \phi(x) \cdot f(t)$$

substitute into SE

$$\cancel{f(t)} \cdot \left[\left(-\frac{\hbar^2}{2m}\frac{d}{dx^2} + U(x)\right)\phi(x)\right] = \cancel{\phi(x)} \cdot i\hbar \frac{d}{dt}(f(t))$$

$$\frac{1}{\phi(x) f(t)}$$

$$\underbrace{\frac{1}{\phi(x)}\left[\underbrace{\left(-\frac{\hbar^2}{2m}\frac{d}{dx^2} + U(x)\right)}_{\text{operator on } \phi(x)}\phi(x)\right]}_{\substack{\text{function of } x \\ \text{no } t}} = \underbrace{\frac{1}{f(t)}\, i\hbar \frac{df}{dt}}_{\substack{\text{function of } t \\ \text{no } x}}$$

how? can vary one, then other needs to vary

only true if both are trivial sol^n's, both must be a constant

$$i\hbar \frac{df}{dt} = f(t) E$$

$$\left(-\frac{\hbar^2}{2m}\frac{d}{dx^2} + U(x)\right)\phi(x) = \phi(x) E \rightarrow \text{time indpt. SE}$$

$$= -\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} + U(x)\,\phi(x)$$