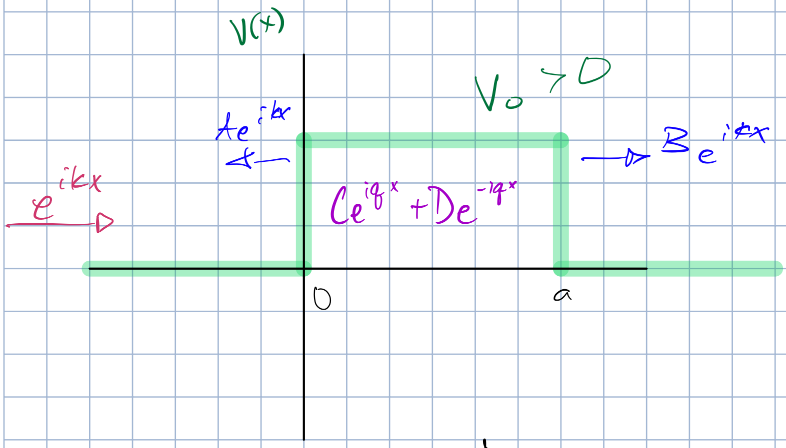


$E > V_0$

$k^2 = \frac{2mE}{\hbar^2}$

$q^2 = \frac{2m}{\hbar}(E - V_0) > 0$

$$T = |B|^2 = \frac{1}{1 + \frac{(k^2-q^2)^2}{4k^2q^2}\sin^2(qa)}$$

classically, would have less energy

slows down then speeds up $F = -\frac{dV}{dx}$

What if $0 < E < V_0$?

classically, would just bounce back

k^2 same, but now $q^2 < 0 \rightsquigarrow q = iQ$ ← real

Sol^{n} inside is now $Ce^{iQx} + De^{-iQx}$

$$\sin(qa) = \frac{e^{iqa} - e^{-iqa}}{2i}$$

$\sinh(x) = \frac{e^x - e^{-x}}{2}$

$$q \to iQ \qquad = \frac{e^{-Qa} - e^{Qa}}{2i} = i\sinh(Qa)$$

$T \longrightarrow$

$\sinh^2(Qa)$

Often in real world, $Qa \gg 1 \longrightarrow \sinh(Qa) \approx \frac{e^{Qa}}{2}$

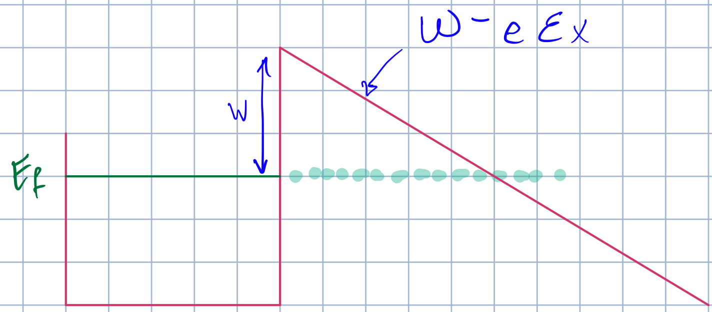
$$\longrightarrow \frac{1}{1 + \frac{(k^2+Q^2)^2}{4k^2Q^2} \cdot \frac{e^{2Qa}}{4}} \simeq \frac{16k^2a^2}{(k^2+Q^2)^2} \underbrace{e^{-2Qa}}_{\text{dominates}}$$

$$e^{-2Qa} = \exp\left(\frac{-2\sqrt{2m(V_0-E)}\,a}{\hbar}\right)$$

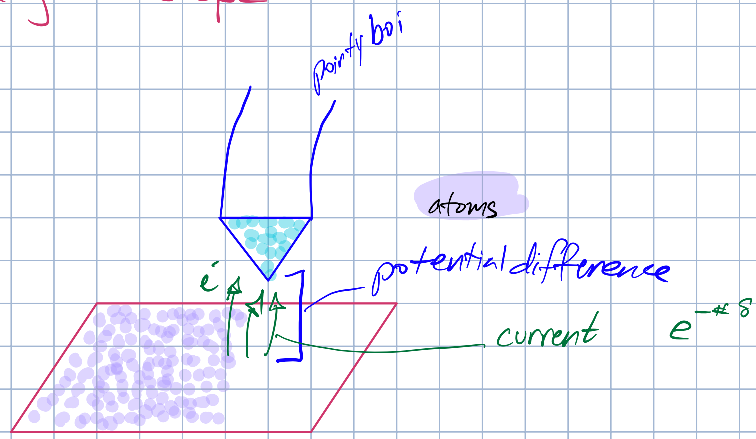
Cold emission of electrons from metals

turn on electric field : $\vec{E} = \mathcal{E}\hat{e}_x$ $\qquad \vec{E} = -\nabla\phi$

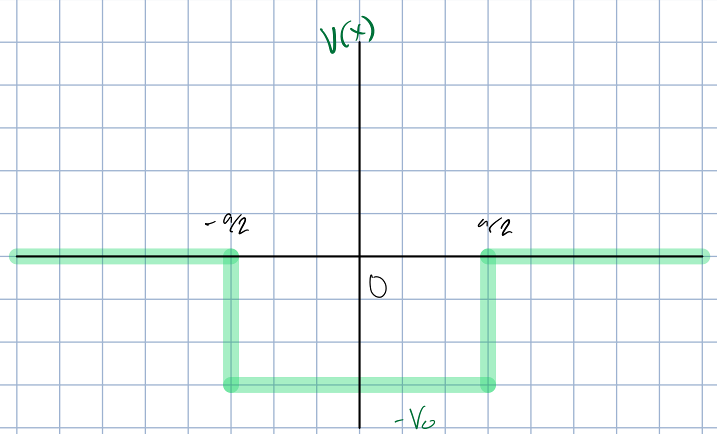
$\phi = -\mathcal{E}x$



Scanning Tunneling Microscope



Bound states of attractive square well



Are there Bound states?

localized, normalizable, sol^n to SE

$E < 0$

$|x| \to a/2$ $\qquad \frac{d^2\phi}{dx^2} = \frac{-2mE}{\hbar^2}\phi = \rho^2\phi$

$\rho^2 = \frac{-2mE}{\hbar^2}$

$V(-x) = V(x)$

$$\phi_{out}(x) = \begin{cases} C_1 e^{\rho x} & x < -a/2 \\ C_2 e^{-\rho x} & x > a/2 \end{cases}$$

$\phi_{in}(x) = A\cos(qx) + B\sin(qx) \qquad q^2 = \frac{2m}{\hbar^2}(E+V_0)$

$\phi_{out} = \phi_{in}$ @ $x = a/2, -a/2$

$\frac{d\phi_{out}}{dx} = \frac{d\phi_{in}}{dx}$ @ " "

$V(-x) = V(x)$

$x \to -x \qquad \hat{p} = -i\hbar \frac{d}{dx} \to -\hat{p}$

$H = \frac{\hat{p}^2}{2m} + V(x)$ is invariant under $x \to -x$

parity operator P

$P|x\rangle = |-x\rangle$

$\langle x|P^+ = \langle -x|$

$\langle x'|P|x\rangle =$

$P^2 = \mathbb{1} \qquad P^2|x\rangle = P|-x\rangle = |x\rangle$

$P = P^+$

$\langle x|P|\psi\rangle = \langle x|P^+|\psi\rangle = \langle -x|\psi\rangle = \psi(-x)$

$P: \psi(x) \to \psi(-x)$

+1 eigenstate → even f^n

−1 eigenstate → odd f^n

$[H, P] = 0$

→ can find energy eigen f^n's that are +1 or −1 eigenstates of P

even sol^n's

$$\phi(x) = \begin{cases} C_1 e^{\rho x} & x < -a/2 \\ A\cos(qx) & -a/2 < x < a/2 \\ C_1 e^{-\rho x} & x > a/2 \end{cases}$$

odd solⁿ's

$$\phi(x) = \begin{cases} C_1 e^{\ell x} & x < -a/2 \\ B\sin(qx) & -a/2 < x < a/2 \\ -C_1 e^{-\ell x} & x > a/2 \end{cases}$$