time indpt SE

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx}\phi(x) + \underbrace{U(x)}_{\text{constant or piecewise constant}}\phi(x) = E\phi(x)$$

$V(x) = U_0$

$$\frac{d^2\phi}{dx^2} = \frac{2m}{\hbar^2}(V_0 - E)\,\phi(x)$$

Solutions : $Ae^{ikx} + Be^{-ikx}$ if $\frac{2m}{\hbar}(V_0 - E) < 0$

→ particle where $E > V_0$

→ $k^2 = -\frac{2m}{\hbar}(V_0 - E_0)$

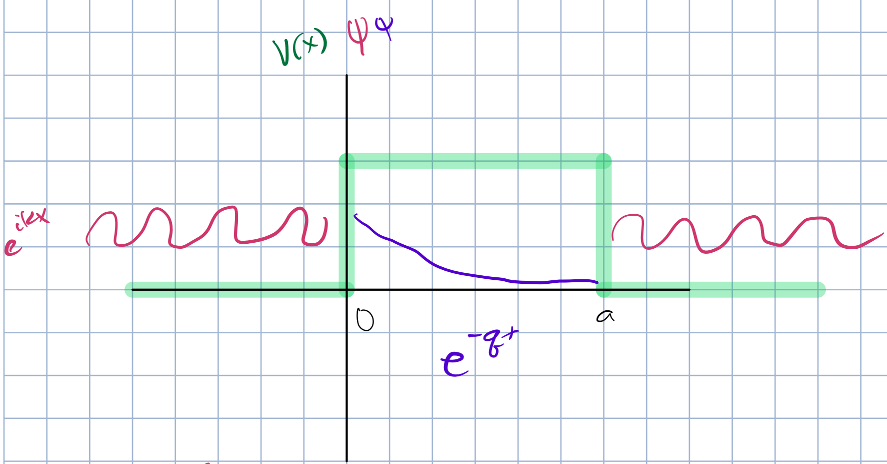
or

$Ce^{iqx} + De^{-iqx}$ if $\frac{2m}{\hbar}(V_0 - E) > 0$

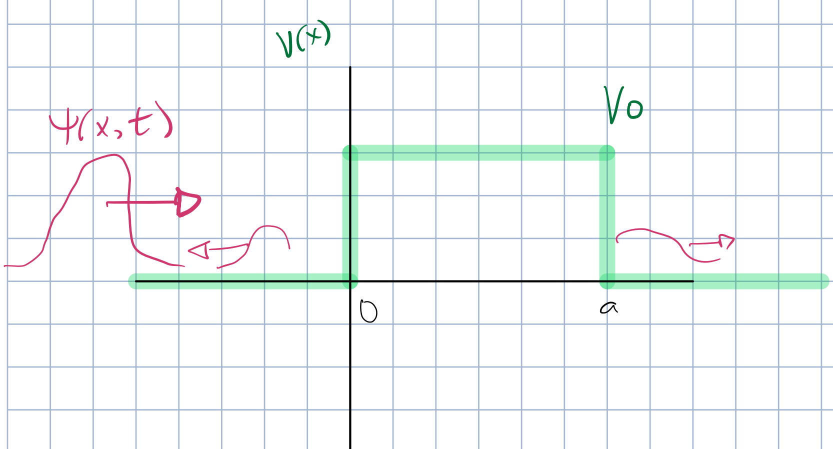
→ " " $E < V_0$

→ $q^2 = \frac{2m}{\hbar}(V_0 - E)$

not allowed for $x \to +\infty$



$0 < E < V_0$



solving Ψ(x,t) is hella hard

sometimes reflected or transmitted

Probability Density

$$P(x,t) = |\psi(x,t)|^2 = |\langle x|\psi(t)\rangle|^2$$

around a region $\int P(x,t)\,dx$

norm. $\int_{-\infty}^{\infty} P(x,t)\,dx = 1$

but locally, probability can change

$\rightsquigarrow \frac{\partial P}{\partial t}$?

$$\longrightarrow \frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial t} \psi^*(x,t)\,\psi(x,t)$$

$$= \frac{\partial \psi^*}{\partial t}\psi + \psi^* \frac{\partial \psi}{\partial t}$$

we also know time dependent SE

$$\longrightarrow \frac{\partial \psi}{\partial t} = \frac{1}{i\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + U(x)\psi\right)$$

$$\frac{\partial \psi^*}{\partial t} = \frac{-1}{i\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 \psi^*}{\partial x^2} + U(x)\psi^*\right)$$

assuming $U(x) = U^*(x)$ needed for $H = H^+$

$$\frac{\partial P}{\partial t} = \frac{\hbar}{2im}\left(\psi\frac{\partial^2 \psi^*}{\partial x^2} - \psi^*\frac{\partial^2 \psi}{\partial x^2}\right)$$

potential goes away

$$= -\frac{\partial}{\partial x}\left(\underbrace{\frac{\hbar^2}{2mi}\left(\psi^*\frac{\partial \psi}{\partial x} - \psi\frac{\partial \psi^*}{\partial x}\right)}\right)$$

$$j(x,t) =$$

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial x} j$$

$$\frac{\partial P}{\partial t} + \frac{\partial j}{\partial x} = 0$$

conservation!

like in E&M

ρ = charge density

j = current density

$$\longrightarrow \frac{\partial \rho}{\partial t} + \vec{\nabla} j = 0$$

$$\int_a^b P(x,t)\,dx \rightarrow \text{prob of it being there}$$

$$\frac{\partial}{\partial t}\int_a^b P(x,t)\,dx = -\int_a^b \frac{\partial j}{\partial x}\,dx$$

$$= j(a) - j(b)$$

$$\psi_p(x) = e^{ipx/\hbar}$$ momentum eigenstate

assume no $U(x)$

what is prob. current of $\psi_p(x)$?

$$j = \frac{\hbar}{2im}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right)$$

$$= \frac{\hbar}{2im}\left(e^{\frac{-ipx}{\hbar}} \cdot \frac{ip}{\hbar} \cdot e^{\frac{ipx}{\hbar}} \dots\right.$$

$$= \frac{\hbar}{2im}\left(\psi^* \frac{\partial \psi}{\partial x} - \left(\psi^* \frac{\partial \psi}{\partial x}\right)^*\right)$$

$$= \frac{\hbar}{2im}\left(e^{\frac{-ipx}{\hbar}} \cdot \frac{ip}{\hbar} \cdot e^{\frac{ipx}{\hbar}} + e^{\frac{ipx}{\hbar}} \cdot \frac{ip}{\hbar} \cdot e^{\frac{-ipx}{\hbar}}\right)$$

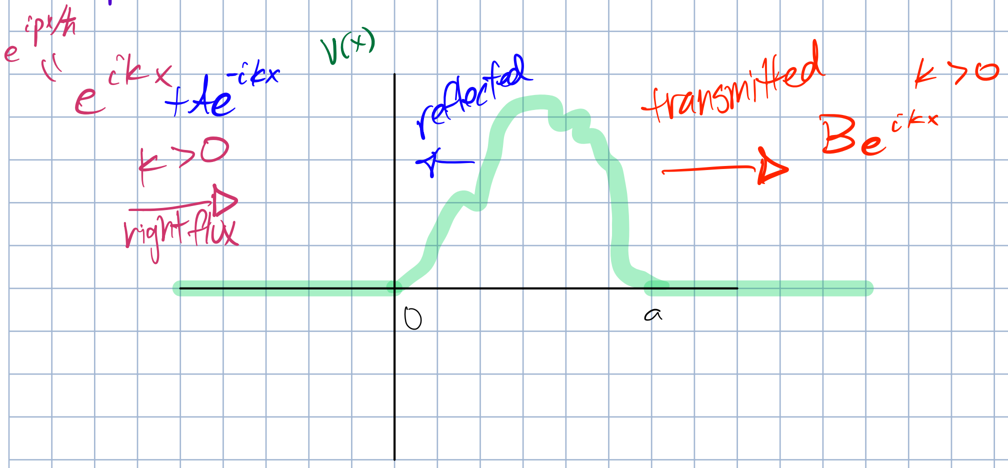
$$= \frac{\hbar}{2im}\left(\frac{ip}{\hbar} + \frac{ip}{\hbar}\right) = \frac{p}{m}$$

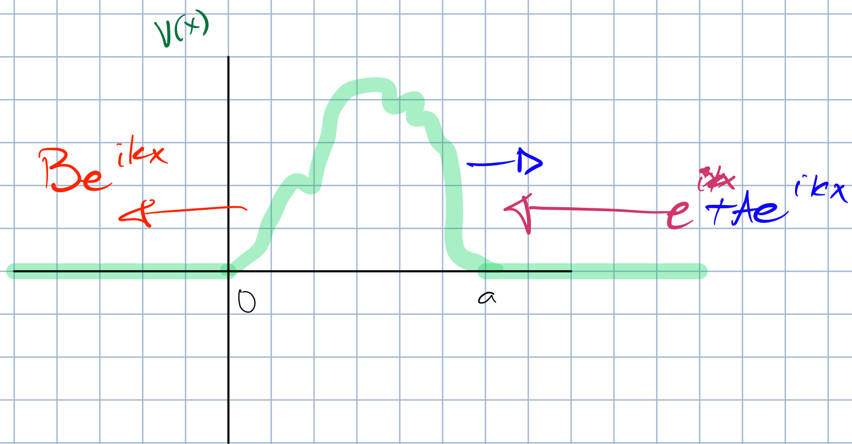
$p > 0$ $\quad j > 0$ flux to the right

prob. density - amt water in river

particles moving to the right

$$\Psi_{\text{wave packet}}(x,t) = \int e^{ipx(\hbar - i\frac{p^2}{2m}t/\hbar)}\, \phi(p)\, dp$$

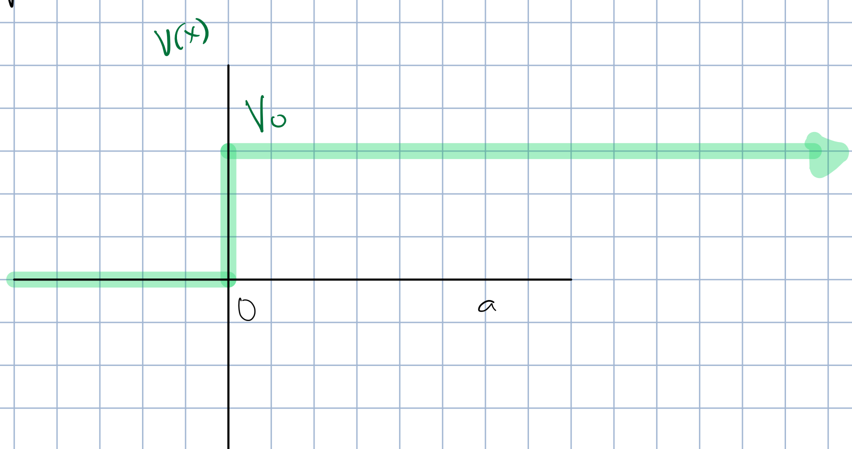




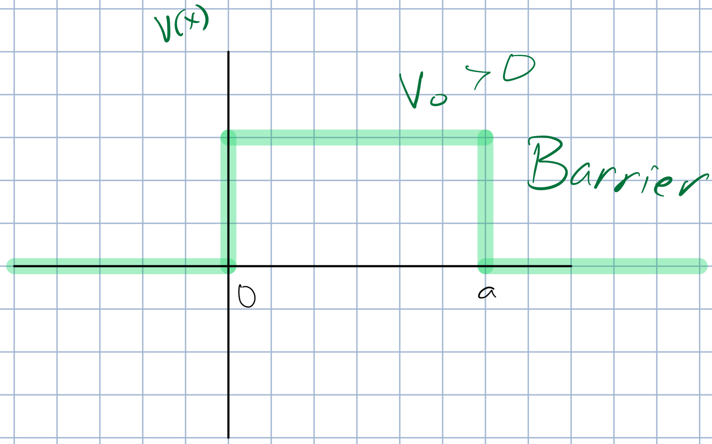
A & B hold physics

Cool examples

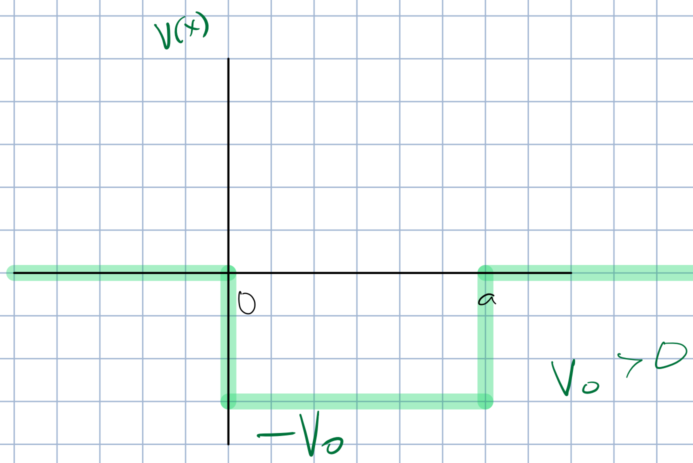
① Step potential



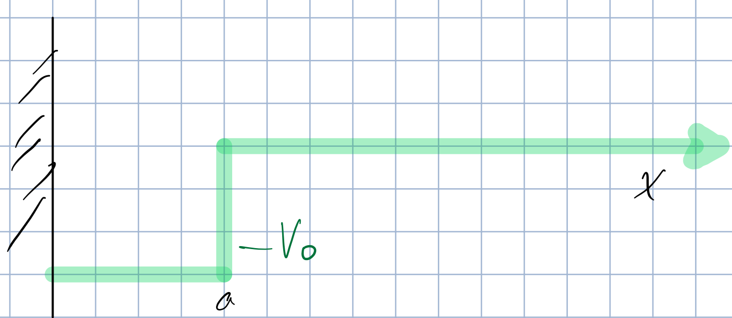
② Repulsive Square Well



③ Attractive Square Well



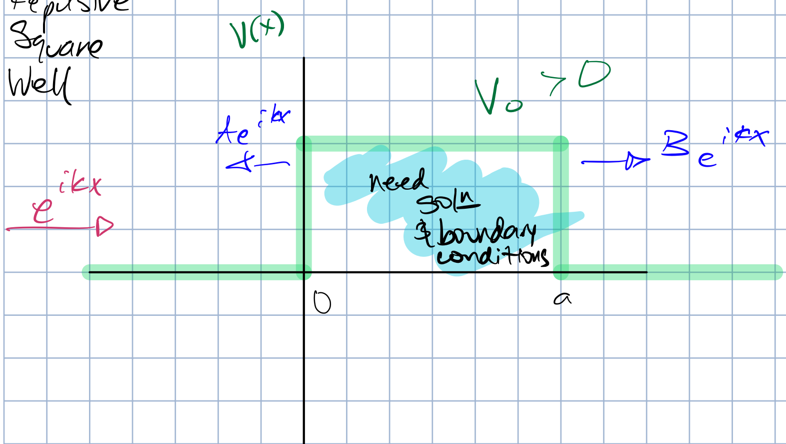
④



spherical coords

Part 2

Repulsive Square Well



No time dependence in $P(x,t)$ (turns out to just change phase)

so $\frac{\partial P}{\partial t} = 0$

$\longrightarrow \frac{\partial j}{\partial x} = 0$

for $x < 0$ $\psi = e^{ikx} + Ae^{-ikx}$

$j = \frac{\hbar}{2mi}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right)$

$j = \frac{\hbar k}{m}\left(1 - |A|^2\right)$

$\psi^* \frac{\partial \psi}{\partial x} = \left(e^{-ikx} + A^* e^{ikx}\right)\cdot\left(ike^{ikx} - Aike^{-ikx}\right)$

$= ik\big(\underbrace{1 - |A|^2}_{\text{real}} + \underbrace{A^* e^{2ikx} - Ae^{-2ikx}}_{\text{complex}}\big)$

with i, pure imaginary pure real

for $x > a$ $\psi = Be^{ikx}$

$j = \frac{\hbar k}{m}|B|^2$ nothing goes other way

but we subtract by complex conjugate

so only real stays

$1 - |A|^2 = |B|^2$

in − reflected = transmitted , conservation

convention: $|B|^2 = T$ = transmission probability

$|A|^2 = R$ = reflection probability

$1 = R + T$

for SE, need sol^n in all regions

$k^2 = \frac{2m(V_0 - E)}{\hbar^2}$

$q^2 = -\frac{2m(V_0 - E)}{\hbar^2} = \frac{2m}{\hbar}(E - V_0)$

I. $x < 0 \longrightarrow \psi_I = e^{ikx} + Ae^{-ikx}$

need to decide if $E > V_0$ (scattering over barrier) . would classically slow down

assume $E > V_0$

II $0 < x < a \longrightarrow \psi_{II} = Ce^{iqx} + De^{-iqx}$

III. $x \geq a \longrightarrow \psi_{III} = Be^{ikx}$

2 conditions

Demand $\psi(x,t)$ is continuous in x potentials are idealized

@ $x = 0, a$

$\psi_I(0) = \psi_{II}(0)$

$\psi_{II}(a) = \psi_{III}(a)$

$$\lim_{\epsilon\to 0}\left(\frac{d\psi}{dx}\Big|_{\epsilon} - \frac{d\psi}{dx}\Big|_{-\epsilon}\right) = \lim_{\epsilon\to 0}\left(\int_{-\epsilon}^{\epsilon}\frac{d^2\psi}{dx^2}\,dx\right) = \lim_{\epsilon\to 0}\left(\int_{-\epsilon}^{\epsilon}\frac{2m}{\hbar^2}\underbrace{(V(x)-E)}_{\text{not singular, so we chillin}}\,dx\right) = 0$$

$$\frac{d\psi_{I}(0)}{dx} = \frac{d\psi_{II}(0)}{dx}$$

$$\frac{d\psi_{II}(a)}{dx} = \frac{d\psi_{III}(a)}{dx}$$

Now algebra & calculus

$x=0$ $\quad\psi\quad$ $1+A = C+D$

$\quad\quad\;\;\psi'\quad$ $ik(1-A) = iq(C-D)$

$x=a$ $\quad\psi\quad$ $Ce^{iqa} + De^{-iqa} = Be^{ika}$

$\quad\quad\;\;\psi'\quad$ $iq(Ce^{iqa} - De^{-iqa}) = ikBe^{ika}$

$$R = |A|^2 = \frac{(k^2-q^2)^2\sin^2(qa)}{4k^2q^2 + (k^2-q^2)\sin^2(qa)}$$

$$T = |B|^2 = \frac{4k^2q^2}{4k^2q^2 + (k^2-q^2)^2\sin^2(qa)}$$

discussion

as $V_0 \to 0$, $\quad q^2 \to k^2$, $\quad R \to 0$, $\quad T \to 1$

$$T = \frac{1}{1 + \frac{(k^2-q^2)^2}{4k^2q^2}\sin^2(qa)}$$

T varying w/E

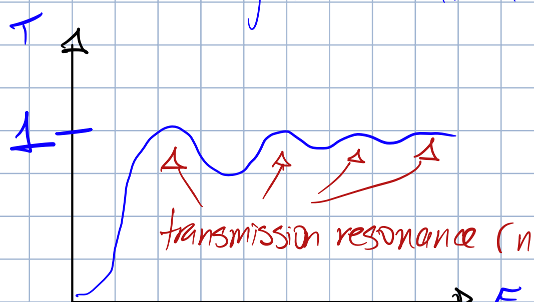
$$q = \sqrt{2m\underline{(V_0-E)}/\hbar}$$

$\sin^2$ varies between 0 & 1

when $\sin^2 qa = 0 \to qa = 0, \pi, 2\pi, \ldots$

$\to T = 1$

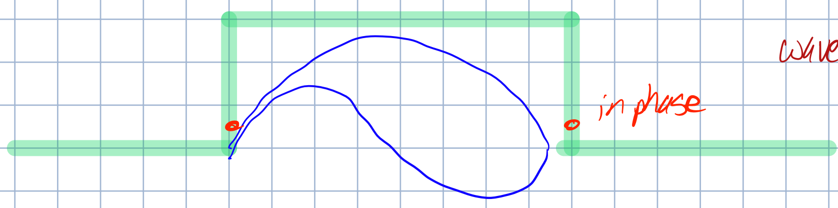
certain energies where all of wave is transmitted



looking e $\quad 0 < x < a \quad e^{iqx}$

$$\lambda = \frac{2\pi}{q}$$

$$qa = n\pi \rightarrow \lambda = \frac{2a}{n}$$



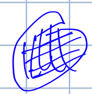
in phase

wave behavior

Ramsauer - Townsend effect

low energy scattering electrons of noble gases

closed shells of electrons



spherical potential