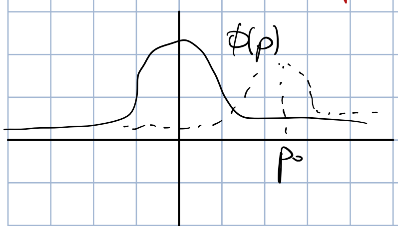
$$\int_{-\infty}^{\infty} e^{-ax^2+bx}\, dx = \sqrt{\frac{\pi}{a}}\; e^{\frac{b^2}{4a}} \qquad a, b \in \mathbb{C},\ \mathrm{Re}[a] > 0$$

We know $\psi_p(x,t) = e^{ipx/\hbar}\, e^{-i(p^2/2m)t/\hbar}$ solves time dependent SE

$$\psi(x,t) = \int_{-\infty}^{\infty} dp\ \phi(p)\, e^{\frac{ipx}{\hbar}}\, e^{\frac{-ip^2t}{2m\hbar}}$$

choose $\phi(p)$ to... ① make $\psi(x,t)$ normalizable ② make calculations easy

Let's choose $\phi(p) = C e^{-\frac{p^2}{2\tilde{\Delta}^2}}$ or $C e^{-\frac{(p-p_0)^2}{2\tilde{\Delta}^2}}$



@ $t=0$, $\psi(x,0) = \int dp\ C e^{-\frac{p^2}{2\tilde{\Delta}^2}} e^{\frac{ipx}{\hbar}} = C\int dp\ e^{-\frac{1}{2\tilde{\Delta}^2}p^2 + \frac{ix}{\hbar}p}$

$\alpha = \frac{1}{2\tilde{\Delta}^2}$ $\quad \beta = \frac{ix}{\hbar}$

$$= C\int dp\ e^{-\alpha p^2 + \beta p} = C \cdot \sqrt{2\pi\tilde{\Delta}^2}\ e^{-\frac{x^2 2\tilde{\Delta}^2}{4\hbar^2}}$$

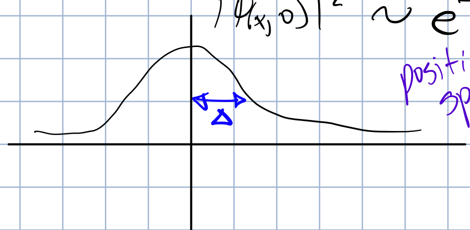
$$= C\tilde{\Delta}\sqrt{2\pi}\ e^{-\frac{x^2\tilde{\Delta}^2}{2\hbar^2}}$$

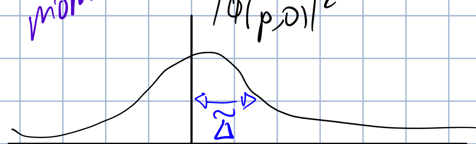
define $\Delta^2 = \frac{\hbar^2}{\tilde{\Delta}^2}$

$\rightarrow \phi(p) = C e^{-\frac{p^2}{2\tilde{\Delta}^2}}$

$\rightarrow \psi(x,0) = C\tilde{\Delta}\, e^{-\frac{x^2}{2\Delta^2}}$

$|\psi(x,0)|^2 \sim e^{-x^2/\Delta^2}$





$$\Delta \tilde{\Delta} = \hbar$$

$\to \phi(p)$, chosen

$$\psi(x,t) = \int_{-\infty}^{\infty} C \, e^{-\frac{p^2}{2\tilde{\Delta}^2} + \frac{ipx}{\hbar} - \frac{ip^2 t}{2m\hbar}}$$

$\to$ from SE

$$= \int_{-\infty}^{\infty} C \, e^{-\left(\frac{1}{2\tilde{\Delta}^2} + \frac{it}{2m\hbar}\right)p^2 + \left(\frac{ix}{\hbar}\right)p}$$

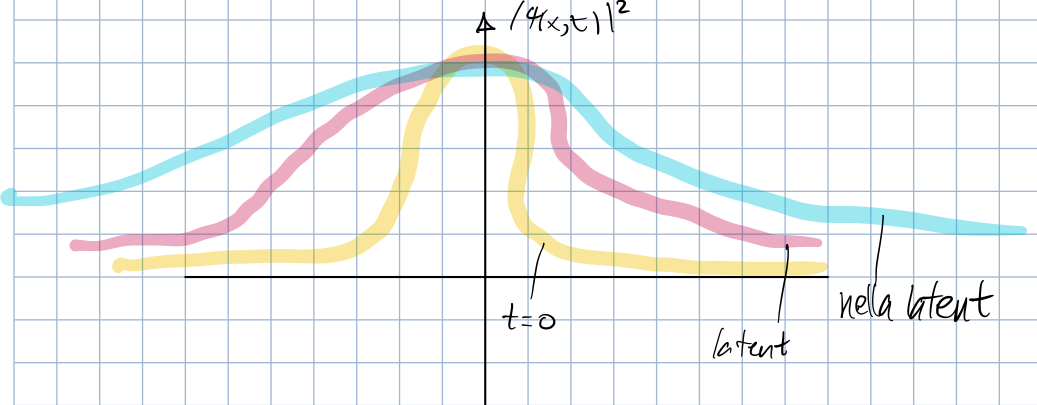
$$= C \cdot \sqrt{\frac{\pi}{\frac{1}{2\tilde{\Delta}^2} + \frac{it}{2m\hbar}}} \cdot e^{-\frac{x^2}{\hbar^2} \cdot \frac{1}{4\left(\frac{1}{2\tilde{\Delta}^2} + \frac{it}{2m\hbar}\right)}}$$

$\alpha = \left(\frac{1}{2\tilde{\Delta}^2} + \frac{it}{2m\hbar}\right)$

$\beta = \frac{ix}{\hbar}$

$$|\psi(x,t)|^2 \propto \exp\left(\frac{-x^2}{\Delta^2(t)}\right)$$

$$\Delta^2(t) = \frac{\hbar^2}{\tilde{\Delta}^2}\left(1 + \frac{t^2 \tilde{\Delta}^4}{m^2 \hbar^2}\right) = \Delta^2(0)\left(1 + \frac{t^2 \hbar^2}{m^2 \Delta(0)^4}\right)$$



spreads out over time

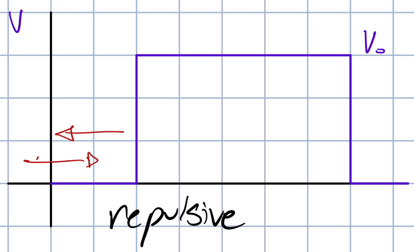
like pouring water on an infinite sheet

1) Particle in a box
discrete set of E_n

2) Free particle
$E = \frac{p^2}{2m}$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right]\phi(x)=E\,\phi(x)$$

piecewise potentials



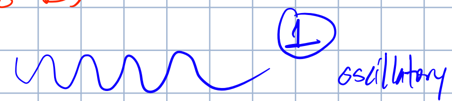
can also be negative

attractive, trap

$$\frac{d^2\phi}{dx^2}=\frac{2m}{\hbar^2}(V_0-E)\,\phi(x)$$

if $\frac{2m}{\hbar^2}(V_0-E)<0 \rightarrow k^2=-\frac{2m}{\hbar^2}(V_0-E)>0$

sol^n is $\phi(x)=Ae^{ikx}+Be^{-ikx}$



if $\frac{2m}{\hbar^2}(V_0-E)>0 \rightarrow q^2=\frac{2m}{\hbar^2}(V_0-E)$

sol^n is $\phi(x)=A'e^{qx}+B'e^{-qx}$