Office: MCP 251

4 sections

Nice grading :) → no psets, lol figure it out

Help w/ E&M & QM

Psets: Wed^s

Topics
- ODEs, PDEs
- Calculus of variations
- Linear vector spaces
- special functions
- integral transforms

— time permitting

HW #1 is up

Grading? lol

live notes going

First ODE

RHB Ch 15, 14

DE of order n $\left[\mathcal{D}_n + f(\vec{x}) \right] y(\vec{x}) = y(\vec{x})$ — $\vec{x}$ can be a vector

differential operator $\mathcal{D}_n \equiv A_{n,1}(\vec{x}) \frac{\partial^n}{\partial x^n} + \dots A_{m,n} \frac{\partial^m}{\partial x^m}$

n is highest order

ex: 1^st ODE

$$\frac{d}{dx}(y(x)) = f(x,y)$$

Linear Forms : ACL $\frac{\partial^n}{\partial x^n}$ enter $\left(\frac{\partial^n f}{\partial x^n}\right)^0$

$[y(x)]'$

General DE : $D[a f_1(x) + b f_2(x)] = a D[f_1] + b D[f_2]$

ODEs : $\vec{x} \to x$, $y(x)$

consequences: n indpt solutions $y_n(x)$

1st order → 1 indpt solution
2nd order → 2 indpt solutions sin , cos

general solution : $y_g(x) = \sum_{m=1}^{n} A_m y_m(x)$ — n is order

n constants of integration
expression of initial conditions , initial constants

y_g satisfies initial conditions

$n=1$ separable cases

$\frac{dy}{dx}$ as fraction

$$\frac{d}{dx} y(x) = y'(x) = \frac{f(x)}{g(y)}$$ known functions

$$\to g(y)\,dy = f(x)\,dx$$

$(x_0, y(x_0)) \to (x, y(x))$

$$\to \int_{y(x_0)}^{y(x)} g(y)\,dy = \int_{x_0}^{x} f(x)\,dx$$

constant of integration

ex: $\dot{v}(t) \equiv a$ const. acceleration

$$\to \int_{v_0}^{v} dv = \int_{t_0}^{t} a\,dt$$

$$\to v(t) - v_0(t_0) = a \cdot t - a t_0$$ $\equiv C$

ex: $\ddot{y} \equiv g$

$$\to \int_{\dot{y}(t_0)}^{\dot{y}(t)} d\dot{y} = g \int_{t_0}^{t} dt$$

$$\longrightarrow \quad \dot{y}(t) - \dot{y}(t_0) = g(t-t_0)$$

again

$$\longrightarrow \quad \int_{y(t_0)}^{y(t)} dy = g\int_{t_0}^{t} t\,dt + c_1 \int_{t_0}^{t} dt$$

$$\longrightarrow \quad y(t) = \frac{1}{2} g t^2 + c_1 t + c_2$$

$n=2 \rightarrow c_1 \,\&\, c_2$

ex drag eq^n $\qquad F = m\dot{v} = mg - \gamma m v^2$

$$\longrightarrow \quad \dot{v} = g - \gamma v^2$$

$$\longrightarrow \quad \frac{dv}{g-\gamma v^2} = dt$$

$$\longrightarrow \quad \int \frac{dv}{g-\gamma v^2} = \int dt$$

$$\longrightarrow \quad \underline{t - t_0} = \frac{1}{2\sqrt{g\gamma}} \ln\left(\frac{g-\gamma v^2}{g-\gamma \underline{v_0}^2}\right)$$

Exact Differential EQNs (14.2.2)

$$d[U(x,y)] \equiv 0$$

$$\underbrace{\frac{\partial U}{\partial x}}_{A(x,y)} \cdot dx + \underbrace{\frac{\partial U}{\partial y}}_{B(x,y)} dy = 0 \qquad \longrightarrow \quad U(x,y) = \text{const.}$$

b/c it's linear

!!!

notice: $\dfrac{\partial A}{\partial y} = \dfrac{\partial^2 U}{\partial x \partial y} = \dfrac{\partial^2 U}{\partial y \partial x} = \dfrac{\partial B}{\partial x}$ $\qquad$ Test

but its a x & y function

solve: ① $\dfrac{\partial U}{\partial x} = A \longrightarrow U = \displaystyle\int_{x_1}^{x_2} A\,dx + F(y)$

② $\dfrac{\partial U}{\partial y} = B = \dfrac{\partial}{\partial y}\displaystyle\int_{x_1}^{x_2} A\,dx + \dfrac{dF}{dy}$

ex: $A = 3x + y$

$B = x$

DE is $\quad xy' + (3x+y) = 0$

$$\frac{\partial A}{\partial y} = 1 \qquad \frac{\partial B}{\partial x} = 1 \qquad \checkmark \quad \text{it is exact}$$

$$U(x,y) = \int_{x_0}^{x} (3x+y)\,dx + F(y) = \frac{3x^2}{2} + xy - \frac{3x_0^2}{2} - x_0 y + F(y) = C$$

$$\longrightarrow \frac{3x^2}{2} + xy + [F(y) - x_0 y] = C - \frac{3x_0^2}{2} \equiv C_1$$

$$B \equiv x = \frac{\partial U}{\partial y} = 0 + x + \frac{dF}{dy} - x_0 \rightarrow F = x_0 y$$

$$U = \frac{3x^2}{2} + xy - \underbrace{\frac{3x_0^2}{2} - x_0 y + x_0 y}_{C_2} = C$$