50/50 130 × 140

Discussion of PS#1
Discussion section assignments — OFC HR: Th 2PM

PS#1

due on Fridays

concepts → what's impt. in RHB & not

Q1 → seperability

Q2 → integrating factor

Q3 → homogeneous defn in 14.2.5

$$y' = \frac{A(x,y)}{B(x,y)} = F(x,y) \quad \text{where } A, B \text{ satisfy} \quad A(\lambda x, \lambda y) = \lambda^n A(x,y) \ldots$$

Q4 → RHS ≠ 0, not homogeneous

y_p soln involves guess — 15.1.4

Last time

$$\ddot{y} + \gamma \dot{y} + \omega_0^2 y = \frac{D}{\mu} \sin(\omega t) = \mathrm{Re}[-i e^{i\omega t}]$$

found $y_p(t)$

if $y = e^{i\omega t}$,

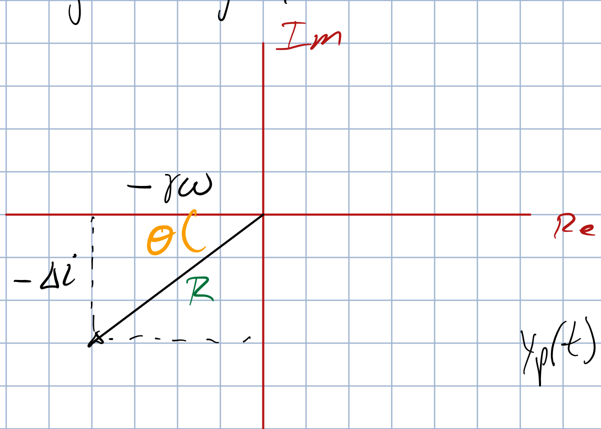
$$y_p(t) = \mathrm{Re}\Big[\frac{D}{\mu}\Big(\overbrace{\frac{-\gamma\omega - i\Delta}{\Delta^2 + \gamma^2\omega^2}}^{A}\Big)\overbrace{e^{i\omega t}}^{e^{i\omega t}}\Big] \qquad \Delta = \omega_0^2 - \omega^2$$

$$= -\frac{D}{\mu}\frac{1}{\Delta^2+\gamma^2\omega^2}\left[\gamma\omega \cos(\omega t) + \Delta \sin(\omega t)\right]$$

Phase variant soln

express $-[\overbrace{\gamma\omega}^{Re} + \overbrace{\Delta i}^{i}]$ as $Re^{i\theta}$

argand diagram



$$R = \sqrt{(\gamma\omega)^2 + \Delta^2}$$

$$\tan\theta = \frac{-\Delta}{-\gamma\omega} = \frac{\mathrm{Im}[\ ]}{\mathrm{Re}[\ ]}$$

$$y_p(t) = \mathrm{Re}\left\{\frac{D}{\mu}\frac{1}{\Delta^2+\gamma^2\omega^2} R e^{i\theta} e^{i\omega t}\right.$$

$$= \frac{D}{\mu}\frac{1}{\sqrt{\gamma^2\omega^2+\Delta^2}}\underbrace{\cos(\omega t + \theta)}$$

$\sin(\omega t + \theta - \pi/2)$

phase difference : $y(t)$ & driving

Series Sol^n RHB 16.2, 16.5

2 other books
Boas : Math Methods in Physical Sciences
good for DE
Shankar : Basic Training in Math
linear vector spaces, QM

2nd order ODE

Have been using linear eq^n's

$$y'' + ay' + by = f(x)$$
$$g(x)$$

$$y_f'' + ay_f' + by_f = f(x)$$

$$y_g'' + a y_g' + b y_g = g(x)$$

Linear def^n: $y_{fg}(x) = y_f + y_g$ solves $y_{fg}'' + a y_{fg}' + b y_{fg} = f(x) + g(x)$

for example

RHS $f(x) = \sum_{n=-\infty}^{\infty} C_n x^n$ ← Laurent Expansion

to make more familiar: let $x = e^{i\theta}$

$$f(\theta) = \sum_{n=-\infty}^{\infty} C_n \underbrace{e^{in\theta}}_{\to\ \cos(n\theta) + i\sin(n\theta)}$$

let $C_{n>0} = \frac{1}{2}(a_n - ib_n)$
$C_{n<0} = \frac{1}{2}(a_n + ib_n)$
$C_0 = \frac{1}{2} a_0$

$$f(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(n\theta) + b_n \sin(n\theta) \right]$$

Return to this later in linear vector spaces RHB ch 17

Series Solutions

$y'' + \omega^2 y = 0$ try $y(x) = \sum_{n=0}^{\infty} C_n x^n$, all reals

don't worry about cases & singular points

$$\sum_{n=0} \left[C_n n \cdot (n-1) x^{n-2} + \omega^2 C_n x^n \right] = 0$$

group powers x : change index of sum $m = n-2$
$n = m+2$

$$\sum_{m=-2}^{\infty} C_{m+2} \cdot (m+2)(m+1)\, x^m + \sum_{n=0}^{\infty} \omega^2 C_n x^n = 0$$

if $m=-2$, 0 $m = -2, -1$ have terms $(m+2)(m+1) = 0$

if $m = -1, 0$

$$\sum_{n=0}^{\infty} \left[C_n (n+2)(n+1) + \omega^2 C_n \right] x^n = 0$$

n are linearly indpt. sol$^{\underline{n}}$'s

x^n indpt f$^{\underline{n}}$'s $\to$ $[\quad]_n = 0$

helps determine C_{n+2} w/ C_n