RHB 14.2, .3, .4 *integrating factor*

Practical variant on exact DE.

any $y'(x) + P(x)\,y(x) = Q(x)$ can be solved

have to make exact by multiply by $e^{p(x)}$
works b/c coefficients are functions of x

if $Q \equiv 0 \longrightarrow$ separable eq^{n} : $\frac{dy}{y} = -P$

$$\int \frac{dy}{y} = -\int_{x_0}^{x} P\,dx \longrightarrow \mu(x) \equiv -\int_{x_0}^{x} P\,dx$$

$$\int \frac{dy}{y} = \mu(x)$$

$\ln(y) = \mu(x)$

$e^{\ln(y)} = e^{\mu(x)}$

$y = e^{\mu(x)}$

integrating factor

$$y(x)\, e^{\mu(x)} = C$$

$$\frac{d}{dx}\left[e^{\mu(x)}\right] = y' e^{\mu} + y e^{\mu} \frac{d\mu}{dx}$$

$$= y' e^{\mu} + y e^{\mu} P$$

$$= e^{\mu}(y' + yP)$$

For $Q \neq 0$, $e^{\mu}(y' + yP) = \frac{d}{dx}\left[y e^{\mu}\right] = e^{\mu} Q(x)$

$$\int d(y e^{\mu}) = \int e^{\mu} Q(x)\,dx$$

$$\longrightarrow y e^{\mu} = \int e^{\mu} Q\,dx$$

$V(t)$ 

$$V(t) = I(t)\cdot R + \dot{I}(t)\cdot L$$

$$\underbrace{\frac{V(t)}{L}}_{Q} = \underbrace{\frac{R}{L}}_{P} I(t) + \dot{I}(t)$$

$$\mu = \int_{t_0}^{t} \frac{R}{L}\,dt = \frac{R}{L}(t - t_0)$$ integration constant

$$I(t) = e^{-\frac{R}{L}(t-t_0)} \int_{t_0}^{t} e^{\frac{R}{L}t} \cdot e^{-\frac{R}{L}t_0} \cdot \frac{V(t)}{L}\, dt$$

$t_0 = 0 \to 1$

w/ $t_0 = 0$

$$I(t) = e^{-\frac{R}{L}t} \int e^{-\frac{R}{L}t} \frac{V(t)}{L}\, dt$$

try $V(t) = \alpha t$

$$I(t) = e^{-\frac{R}{L}t} \frac{\alpha}{L} \int e^{-\frac{R}{L}t}\, t\, dt \to e^{\frac{R}{L}t}\left[\frac{t}{R/L} - \frac{1}{(R/L)^2}\right]_0^t$$

$$I(t) = \frac{\alpha R^2}{L^3}\left[\frac{R}{L}t - 1 + e^{-R/L\, t}\right]$$

Use recipe? rest of chapter good reference

RHB 15.1.1 2^{nd} order ODE

Start by examining "homogeneous," eq^n's

$$y''(x) + A\, y'(x) + B\, y(x) = 0 \qquad A, B \to \text{constants}$$

Factorization: $D \equiv \frac{d}{dx}$, $y'' + Ay' + By = (D+a)(D+b)\, y$

$$= [D^2 + (a+b)D + ab]\, y$$

$A = a+b$
$B = a \cdot b$

linear $\to$ $\underbrace{(D+a)(D+b)\, y}_{=0\,?} = \underbrace{(D+b)(D+a)\, y}_{=0\,?} \equiv 0$

$(D+a)\, y = y'' + ay = 0$ & $(D+b)\, y = y'' + by = 0$

$\to \int_{y(x_0)}^{y(x)} \frac{dy}{y} = \int_{x_0}^{x} -a\, dx$

$y_a(x) = e^{-ax} C_a \quad \to \quad y_b(x) = e^{-bx} C_b$

general sol^n: $y_g = C_a e^{-ax} + C_b e^{-bx}$

what if $a = b$? $\to$ only one e^{-ax}? no

trick: $(D+a)\, y = u(x) \Rightarrow (D+a)\, u(x) = 0$ is full DE

$\rightarrow\ u(x) = C e^{-ax} \implies (D+a)\, y = C e^{-ax} = y' + ay$

trick#2: $y' + ay = e^{-ax} (y e^{ax})'$

$\rightarrow (y e^{ax})' = C \xrightarrow{\text{integrate}} y e^{ax} = C(x - x_0) = Cx + \delta$

2 indpt. sol^{n}'s : $x e^{-ax}, e^{-ax}$

not needed for homogeneous eq^{n} . $y \rightarrow y - \frac{C}{B} \equiv \tilde{y}$

$y'' + Ay' + By \boxed{+ C} = Q(x)$

linear independence (RHB 15)

consider $n=3$ DE : $(D+a)(D+b)(D+c)\, y(x) = 0$

sol^{n}'s $e^{-ax}, e^{-bx}, e^{-cx}$

$e^{-ax} \neq C_1 e^{-bx} + C_2 e^{-cx}$

Test : no constants can be found s.t. $\sum_{n=1}^{m} C_m y_m = 0$

"Wronskian" $\equiv$ $\det \begin{bmatrix} c_1 y_1 & c_2 y_2 & \cdots & c_n y_n \\ c_1 y_1' & \cdots & & c_n y_n' \\ \vdots & & & \\ c_1 y_1^{(n-1)} & & & c_n y_n^{(n-1)} \end{bmatrix} \neq 0$

ex: $n=2$ e^{ax}, e^{bx}

$\det \begin{bmatrix} c_1 e^{ax} & c_2 e^{bx} \\ c_1 a e^{ax} & c_2 e^{bx} \end{bmatrix} = C_1 C_2 e^{ax} e^{bx} (b-a)$

b/c $a \neq b$, e^{ax} & e^{bx} are indpt.

Monday : 2^{nd} order eq^n's still
complex exp.