Separable ODEs

drag

$$F = m\ddot{x} = -mg - m\gamma\dot{x}$$

$$\ddot{x} + \gamma\dot{x} + g = 0$$

$$d\dot{x} = \frac{d\dot{x}}{dt} = -(\gamma\dot{x}+g) \rightarrow \quad \frac{d\dot{x}}{\gamma\dot{x}+g} = -dt$$

$x_0 \rightarrow IC \qquad \dot{x} = \frac{g}{\gamma}\left(e^{-\gamma t} - 1\right) = \frac{dx}{dt}$

spring w/ drag?

hooke's constant

$$F = m\ddot{x} = -mg - m\gamma\dot{x} + kx$$

ODE : $\ddot{x} + \gamma\dot{x} + g + \underbrace{\frac{k}{m}}_{=\omega_0^2}x = 0$

let $y = x + \frac{g}{\omega_0^2}$

$$\ddot{y} + \gamma\dot{y} + \omega_0^2 y = 0$$

constant coefficients? try $y(t) = Ae^{\alpha t}$

$$\rightarrow \underbrace{(\alpha^2 + \gamma\alpha + \omega_0^2)}_{=0} Ae^{\alpha t} = 0$$

$$\alpha = -\frac{\gamma}{2} \pm \underbrace{\sqrt{\omega_0^2 - \frac{\gamma^2}{4}}\, i}_{\equiv \omega}$$

$$y(t) = \left[Ae^{i\omega t} + Be^{-i\omega t}\right] e^{-\frac{\gamma}{2}t}$$

or $C\cos(\omega t) + D\sin(\omega t)$ $\qquad$ C, D satisfy IC.

Driving force

exponentials

$\ddot{y}_p + \gamma\dot{y}_p + \omega_0^2 y_p = De^{\alpha t},\ De^{i\alpha t},\ \sin(\alpha t) \ldots$

now $y(t) = Ae^{i\alpha t}$

2 part sol$^{\underline{n}}$

a) Get A from $\left[-\alpha^2 + i\gamma\alpha + \omega_0^2\right]\overset{\star}{A}\cancel{e^{i\alpha t}} = D\cancel{e^{i\alpha t}}$

b) use homogeneous eq^{n} $\ddot{y}_c + \gamma \dot{y}_c + \omega_0^2 y_c = 0$

to get $y_p(t)$ w/ 2 constants

Nonconstant coefficients

polynomials

$$\ddot{y} + P_m(x)\, y' + P_n(x)\, y = 0$$

try power series $y = x^s \sum_{n=0}^{\infty} c_n x^n$

group powers of x^n $\rightarrow$ $\sum [\ldots] x^n = 0$

recursion $c_{n+2} = \ldots c_n$

PDEs

sep. of variables

$F(x,t) = X(x)\, T(t) \rightarrow$ 2 ODEs & "eigen values"

Change of Basis

R+B ch 8, 13

use basis to describe object in LVS: $|v\rangle = \sum_{i=1}^{N} V_i \underbrace{|i\rangle}_{\text{basis objects}}$

if basis orthonormal, then

$$\langle j|v\rangle = \sum_{i=1}^{N} \langle j\, V_i|i\rangle = \sum_{i=1}^{N} V_i \underbrace{\langle j|i\rangle}_{\delta_{ji}}$$

$$V_j = \langle j|v\rangle$$

$$\vec{v} = \sum_{i=1} V_i \hat{e}_i \,, \qquad V_i = \hat{v} \cdot \hat{e}_i$$

What if I have bases Σ, Σ' $\leftarrow$ some rotation

$$|v\rangle = \sum_i V_i \hat{e}_i = \sum_i V_i |i\rangle = \sum_i V_i' |j'\rangle$$

rotations

object, not basis dependent

are basis dependent

then $v_j' = \langle j' | v \rangle$

now, I can expand $\langle j'| = \sum_k^N \langle j'|k\rangle\langle k|$

identity

often stated: " $\sum_k |k\rangle\langle k| = \mathbb{1}$ " $= \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix}$

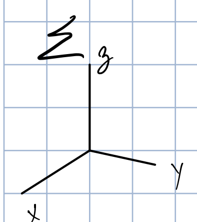
$v_j' = \sum_k \langle j'|k\rangle\langle k \underbrace{|v\rangle}_{\sum v_i |i\rangle} = \sum_k \sum_i \overbrace{\langle j'|k\rangle}^{\text{prime w/unprime who knows?}} \overbrace{\langle k|i\rangle}^{\text{orthonormal } \delta_{ki}} v_i$

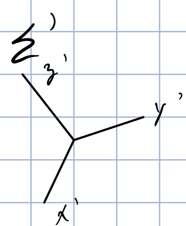
$v_j' = \sum_{i=1}^N \underbrace{\langle j'|i\rangle}_{M_{ji}\text{ - matrix}} v_i$ w/ $k=i$

Similarity transformation

$\vec{v}\,' = \underset{\sim}{M} \cdot \vec{v}$

$$\underset{\sim}{M} = \begin{array}{c|c} & i \rightarrow \\ \hline j \downarrow & \begin{bmatrix} \langle 1'|1\rangle & \langle 1'|2\rangle & \cdots & \langle 1'|N\rangle \\ \langle 2'|1\rangle & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \langle N'|1\rangle & \cdots & & \langle N'|N\rangle \end{bmatrix} \end{array}$$





direction cosines

$\langle 1'|2\rangle = \hat{x}' \cdot \hat{y}$

dot product

project old $\hat{y}$ onto new $\hat{x}$

Matrices can be LVS objects too

8.3, 8.4

$\vec{v} = (a, b) \rightarrow [a, b]$ or $\begin{bmatrix} a \\ b \end{bmatrix}$ 1×2 or 2×1

all associative rules hold

inner product for matrix multiplication

$A \cdot B \neq B \cdot A$

consider LVS of 2×2

basis $V = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$