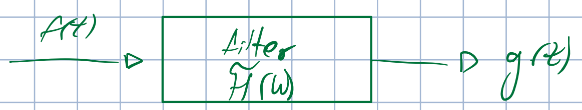
Fourier Convolution: $P(x) * Q(x) = \int_{-\infty}^{\infty} P(x-x')\,Q(x')\,dx' = \int_{-\infty}^{\infty} Q(x-x')\,P(x')\,dx'$

$$\left[\widetilde{P \cdot Q}\right](\omega) = \frac{1}{\sqrt{2\pi}}\, \tilde{P}(\omega) * \tilde{Q}(\omega)$$

One way to use: "Transfer fn"



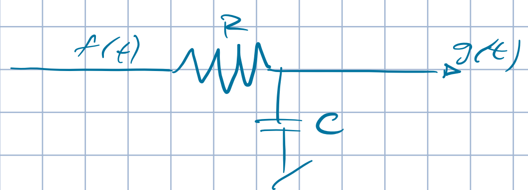
check for "non-filter": $\tilde{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, e^{i\omega t} f(t)$

$$g(t) = f(t) = \frac{1}{\sqrt{2\pi}} \int d\omega\, e^{-i\omega t} \tilde{f}(\omega)$$

if filter modifies $\tilde{f}(\omega)$: $\tilde{g}(\omega) = \tilde{f}(\omega) \cdot \tilde{H}(\omega)$

Output signal: $g(t) = \frac{1}{\sqrt{2\pi}}\, f(t) * H(t) = \int_{-\infty}^{\infty} f(t-t')\,H(t')\,dt'$

example:



Kirchoff: $\dot{g} = \frac{\dot{Q}}{C} = \frac{I}{C} = \frac{V_R/R}{C} = \frac{f-g}{RC}$

define $\alpha = \frac{1}{RC}$ $\qquad \dot{g} + \alpha g = \alpha f$

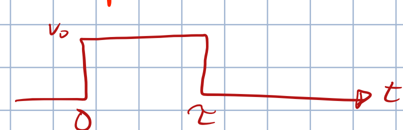
take F.T. $\longrightarrow$ $i\omega\, \tilde{g} + \alpha \tilde{g} = \alpha \tilde{f}$

$$\longrightarrow \tilde{g} = \tilde{f} \cdot \underbrace{\frac{\alpha}{\alpha + i\omega}}_{\tilde{H}(\omega)}$$

get $H(t)$ from $\tilde{H}(\omega) = \frac{\alpha}{\alpha + i\omega}$ $\xrightarrow{\text{tables}}$ $H(t) = \sqrt{2\pi}\, \alpha\, e^{-\alpha t}\, \underbrace{u(t)}$

Step fn

Choose input signal $\qquad f(t) =$



$g(t) = \frac{1}{\sqrt{2\pi}}\, f(t) * H(t)$

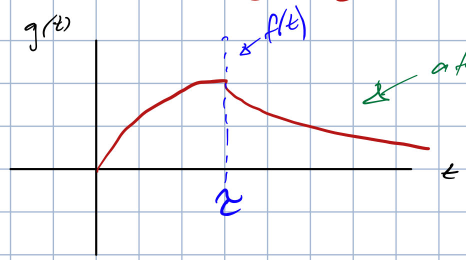
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty} f(t')\, H(t-t')\, dt'$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t')\, \sqrt{2\pi}\, \alpha\, e^{-\alpha(t-t')}\, \underbrace{u(t-t')}_{\to\, 0 \text{ for } t-t' < 0}$$

$$= V_0 \alpha \cdot \begin{cases} t < \tau \;\to\; H(t-t') = 0 \text{ for } t' > 0 \;\to\; \int_0^t e^{-\alpha(t-t')}\, dt' \\ t > \tau \;\to\; \int_0^\tau e^{-\alpha(t-t')}\, dt' \end{cases}$$

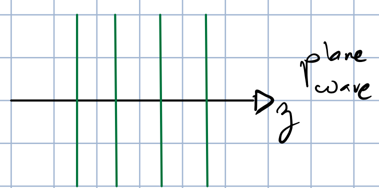
$1/\alpha$ minus sign b/c change of range of integration

$$= V_0 \alpha \frac{1}{\alpha} \begin{cases} e^{-\alpha \cdot 0} - e^{-\alpha t} \\ e^{-\alpha(t-\tau)} - e^{-\alpha t} \end{cases} \begin{aligned} &= V_0 (1 - e^{-\alpha t}) && 0 < t < \tau \\ &= V_0 e^{-\alpha t} (e^{\alpha \tau} - 1) && t > \tau \end{aligned}$$

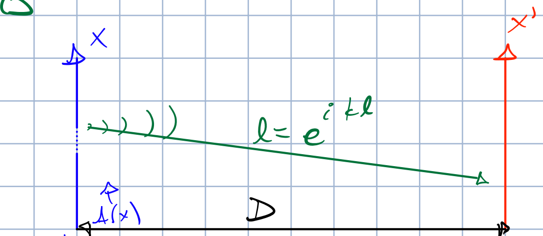


after point, electronic won't read data
ex. HDMI greater than 1.5m

Fourier Transform in Optics



$e^{i(kx-\omega t)}$



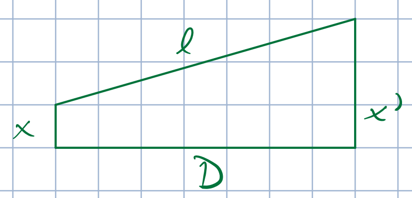
aperture (filter)

amplitude $A(x)$ (all have same phase) coming out

screen

amplitude & phase $\Psi(x',t)$

geometry:



$$l = \sqrt{D^2 + (x-x')^2} \underset{D \gg x, x'}{\simeq} D\left[1 + \frac{1}{2} \frac{(x-x')^2}{D^2}\right]$$

let $x' \gg x \;\to\; l \simeq D\left(1 + \frac{1}{2} \frac{2xx'}{D^2}\right) = D + \frac{xx'}{D}$

Signal @ screen : $\int_{\text{aperture}} A(x)\, e^{i(kl-\omega t)}\, dx$

Far Field

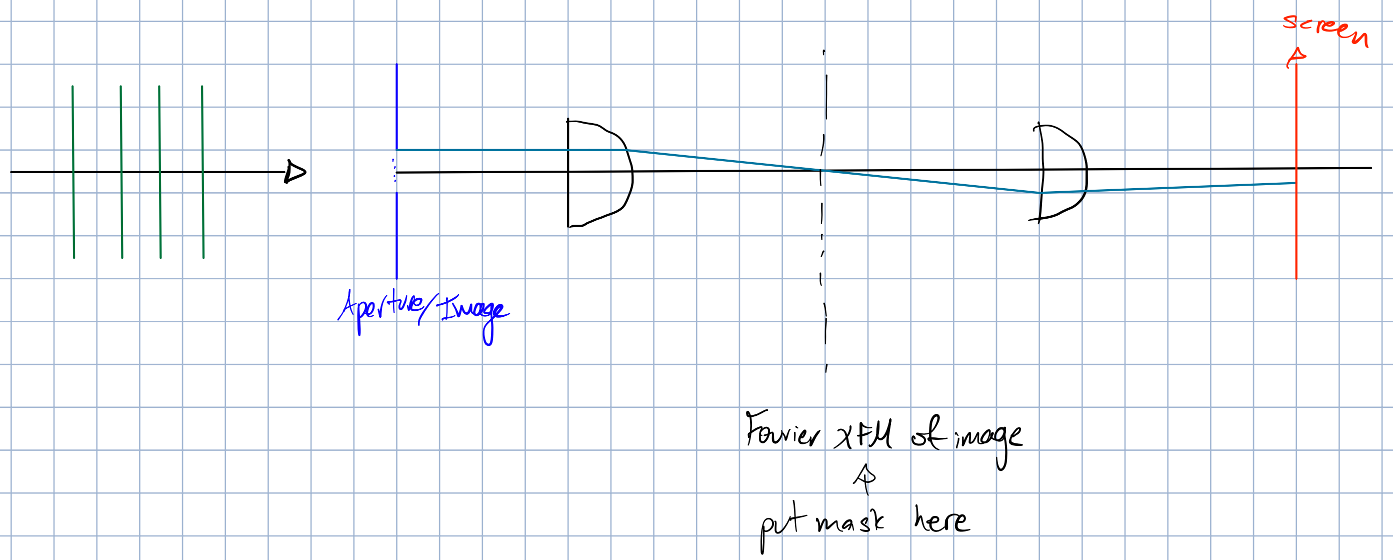
$$\Psi(x',t) \simeq e^{-i\omega t} \int A(x)\, e^{ik(D + \frac{xx'}{D})}\, dx$$

$$= e^{i(kD-\omega t)} \int A(x)\, e^{i\left(\frac{kx'}{D}\right)x}\, dx$$

fourier "frequency"

$$= \sqrt{2\pi}\, e^{i(kD-\omega t)} \cdot \tilde{A}\left(k \frac{x'}{D}\right)$$

Works for lens too! (ideal)



F.T. doesn't always converge :(