Another way to think of coupled DE's

$$\dot{\vec{x}} = \frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \underline{M}\,\vec{x}$$

$\Leftrightarrow$ can diagonalize by changing basis: $\vec{x} = S\vec{x}'$ $\longrightarrow$ $\underline{M}' = \underline{S}^T \underline{M}\underline{S}$

$$\dot{x}_i = M_{ii}'\, x_i$$

solved by $x_i' = C e^{\lambda t}$

can use ansatz $\vec{x} = \vec{A} e^{\lambda t}$ & plug in

$$\dot{\vec{x}} = \lambda \vec{A} e^{\lambda t} = \underline{M}\,\vec{x}$$ eigen value problem

get λ's $\det[\underline{M} - \lambda \mathbb{1}] = 0$ gives λ_i & eigenvectors $\vec{v}_i$

in general: $x_i(t) = \sum_n C_n^{(i)} e^{\lambda_n t}$

plug in to solve for $C_n^{(i)}$

for each n, $C_n^{(i)} \propto V_n^{(i)}$

$$\vec{x}(t) = \sum_n C_n \vec{V}_n e^{\lambda_n t}$$

proof $\dot{x}_i(t) = \sum_n \lambda_n C_n \cancel{e^{\lambda_n t}} = \sum_j M_{ij} \underline{x_i} = \sum_j M_{ij} \sum_n C_n^{(i)} \cancel{e^{\lambda_n t}}$ each are indept

for each $e^{\lambda_n t}$: $\lambda_n C_n^{(i)} = \sum_j M_{ij} C_n^{(i)}$

$\vec{C}_n$: $\lambda_n \vec{C}_n = \underline{M}\,\vec{C}_n$ $\vec{C}_n$ are eigenvectors of $\underline{M}$

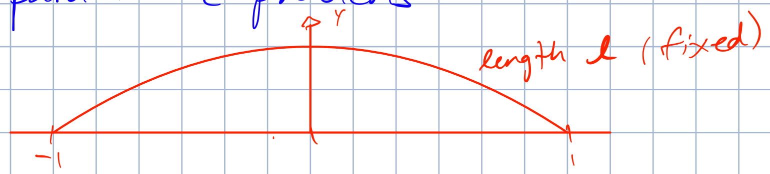
Calc of Variations

1) set up problem to get $J = \int_a^b F(y, y'; z, z'; x)\,dx$

2) Use E-L or Beltrami to get a DE ... solve

if constraints: $F \to \tilde{F} = F + \lambda G$

① Isoparametric problems



what $y(x)$ maximizes area?

$$J = \int_{-1}^{1} y(x)\,dx \qquad F = y(x)$$

$$\ell = \int ds \longrightarrow G = \sqrt{1+y'^2} - \ell$$

$$\tilde{F} = F + \lambda\sqrt{1+y'^2} = \tilde{F}(y, y') \qquad \text{no explicit } x$$

Beltrami: $\tilde{F}_x = 0 = \frac{d}{dx}\left[\tilde{F} - y'\tilde{F}_{y'}\right] \longrightarrow \left(y + \lambda\sqrt{1+y'^2}\right) - \lambda\frac{y'^2}{\sqrt{1+y'^2}} = C$

$$\longrightarrow y - c = \lambda\left(\frac{y'^2}{\sqrt{1+y'^2}} - \sqrt{1+y'^2}\right) = \frac{\lambda}{\sqrt{1+y'^2}}\left(y'^2 - (1+y'^2)\right)$$

$$= \frac{\lambda}{\sqrt{1+y'^2}}(-1) = -\frac{\lambda}{\sqrt{1+y'^2}}$$

$$\longrightarrow (y-c)^2(1+y'^2) = \lambda^2$$

$$y'^2 = \frac{\lambda^2 - (y-c)^2}{(y-c)^2} \qquad \text{seperable! :)}$$

$u = y - c \qquad \frac{du}{dy} = -c \qquad dy = -\frac{du}{c}$

$$\int \frac{u\,du}{\sqrt{\lambda^2 - u^2}} = \int dx$$

math

$$\longrightarrow \lambda^2 - (y-c)^2 - (x-c_2)^2 = \text{circle!}$$

Fermat's Principle: E-M radiation takes route minimizing travel time

$$J = \Delta t = \int_{\text{start}}^{\text{end}} dt = \int \frac{ds}{v} = \frac{1}{c}\int n(x,y)$$

$\frac{1}{c}$: light in vaccum; $n(x,y)$: index of refraction

in ionosphere: $n(y) \approx \sqrt{1-by}$ (high frequencies)

$$F = \sqrt{1-by}\sqrt{1+y'^2} \longrightarrow F(y, y') \quad \text{no } x$$

Beltrami: $= F_y - y'F_{y'} = \sqrt{1-by}\left(\sqrt{1+y'^2} - y'\frac{y'}{\sqrt{1+y'^2}}\right)$

$$= \frac{\sqrt{1-by}}{\sqrt{1+y'^2}} = D$$

$$D^2 (1+y'^2) = 1-by$$

$$D^2 y'^2 = 1 - D^2 - by$$

$$\to \pm D \int \frac{dy}{\sqrt{(1-D^2)-by}} = \mp \frac{D}{b} \int \frac{dw}{\sqrt{w}}$$

$$= \mp \frac{2D}{b} \sqrt{1-D^2-by} = \int dx = x - D_2$$

$$y = \frac{-b}{4D^2}(x-D_2)^2 + \frac{1-D^2}{b}$$

$$\quad m \qquad (x-h)^2 \quad + \quad k$$

parabola

short wave high frequency "skipping"