ODEs

seperate & integrate

do limits correctly

$\int$

"homogeneous" (RHS = 0) 2nd ODE : $(D-a)(D-b)\,y(x) = 0$

RHS $\neq 0$: try exponentials $e^{\alpha x}$

$\longrightarrow$ general sol^n is both particular & "homogeneous" $\rightarrow$ $y_g(x) = y_p(x) + y_h(x)$

exp, sin, cos sol^n's & 2 I.C

all else fails: power series $y = x^s \sum_{n=0}^{\infty} A_n x^n$ $\rightarrow$ use independence of x^n

PDEs

seperation of variables technique : $y(x,t) = R(x) \cdot T(t)$

use separtion constant (eigenvalues determined by B.C.)

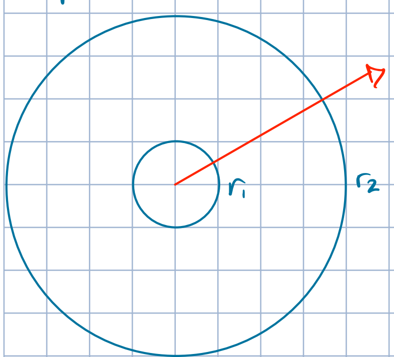
$\sin(\lambda L) = 0 \rightarrow \lambda_n = \frac{\pi n}{L}$

find coefficients of expansion using orthagonality of f^n's

sin, cos

Sep. variables for radial f^n's satisfying $\nabla^2 R = 0$

3D $\rightarrow$ 1D



can't just have $R = \frac{u(r)}{r}$

origin = 0 b/c R blows up $\rightarrow$ r_1 B.C.

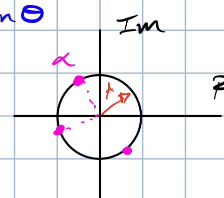
B.C. $R(r-r_1) = 0$

euler : $e^{i\theta} = \cos\theta + i\sin\theta$

$e^{i\theta} = \cos(-\theta) + i\sin(-\theta) = \cos\theta - i\sin\theta$

$x = Ae^{i\theta}$, $x^{\frac{1}{\alpha}} = (Ae^{i\theta})^{1/\alpha} = A^{1/\alpha} \cdot e^{i(\frac{\theta}{\alpha})}$

find α root



LVS

rules : $|w\rangle$ & $\langle v|$, $\langle v|w\rangle = \langle w|v\rangle^*$

basis expansion

golden rule : $\mathbb{1} = \sum_{i=1}^{N} |e_i\rangle\langle e_i|$

linear operators

$\mathcal{L}_{ij} = \langle e_i|\mathcal{L}|e_j\rangle$

$\mathcal{L}$ acts on $|e_j\rangle$
creates different, take $\langle e_i|$ with that

transformation of bases

$$|e_j'\rangle = \sum_{i=1}^{N} |e_i\rangle \underbrace{\langle e_i|e_j'\rangle}_{\text{similarity transform}}$$

similarity transform : $S_{ij} = \langle e_i|e_j'\rangle$

$(S)^T = S_{ji} = \langle e_i'|e_j\rangle$

$\vec{x} = \underline{S}\,\vec{x}'$

$\vec{x}' = \underline{S}^{-1}\,\vec{x}$

Eigenvectors & Eigenvalues

$\underline{M}$ operator

$\underline{M}\,\vec{v}_n = \lambda_n \vec{v}_n$

# of dimensions in $\underline{M} \overset{\text{in general}}{=}$ # n groupings

eigenvectors

for hermitian or unitary $\underline{M}$: $\underline{S} = \begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{bmatrix}$

$\underline{S}^T \underline{M}\,\underline{S} =$ diagonal $= \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{bmatrix}$

Creating orthonormal basis : Gram Schmidt

$|f(x)\rangle$ & function bases

e^{inx} or $\sin$ & $\cos(nx)$ Fourier Series

Fourier Series (only for periodic f^{n}s $f(x+\text{period}) = f(x)$)

$$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{i\omega_0 x n}, \quad \omega_0 = \frac{2\pi}{\text{Period}}$$

$$f(x) = C_0 + \sum_{n=1}^{\infty} \left[A_n \sin(n\omega_0 x) + B_n \cos(n\omega_0 x) \right]$$

period: $0 \to L$

use $\int_0^L \sin(n\omega_0 x)\sin(m\omega_0 x)\,dx = \frac{L}{2} \delta_{mn}$

$\to A_n = \frac{2}{L} \cdot \int_0^L dx \cdot f(x) \cdot \sin(n\omega_0 x)$

Extrema Problems

Undetermined multipliers

$f(x,y,z) \to f - \lambda\, g(x,y,z) \equiv \tilde{f}(x,y,z)$

multiplier, ~~not~~ not eigenvalue

constant, constraint

find stationary points $\frac{\partial \tilde{f}}{\partial x_i} = 0$

bring info. of constraint into problem

Calculus of Variations

determine quantity to extremize $J[y(x)] = \int_a^b f(y', y; x)\, dx$

fixed endpoints

determine best indpt. variable

ds as f^n of dx only

plug into E-L or Beltrami

E-L: no explicit y

Beltrami: no explicit x

if there are constraints: $G(x,y,\ldots) = 0$

$F \to F + \lambda G$

solve resulting DEs

Not on Final

Integral Transforms

Fourier XFM $\Longrightarrow$ frequency spectrum nice properties for solving DEs

Laplace XFM $\Longrightarrow$ solve DEs, IC embedded