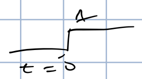
Laplace XFM

$$\tilde{f}(s) \equiv \int_0^\infty dt\, f(t)\, e^{-st} \qquad \mathrm{Re}[s] > 0$$

s is analog of ω, but has no physical meaning

inverse XFM: $f(t) = \frac{1}{2\pi i} \int_{\alpha - i\infty}^{\alpha + i\infty} ds\, \tilde{f}(s)\, e^{st}$

ex: $\tilde{\delta}(t) = \int_0^\infty dt\, \delta(t)\, e^{-st} = 1$



$\tilde{u}(t) = \int_0^\infty dt\, u(t)\, e^{-st} = \frac{1}{s}$

$\widetilde{e^{-at}} = \int_0^\infty dt\, e^{-(s+a)t} = \frac{1}{s+a}$

table

Convolution: $f(t) * g(t) = \int_0^t dt'\, f(t')\, g(t-t')$ (laplace

$$\widetilde{[f * g]} = \tilde{f}(s) \cdot \tilde{g}(s)$$

Laplace often f^{n}'s that make Fourier XFM $\to \infty$

R-C Circuit:

Usual kirchoff : V-drops $\quad V = V_R + V_c = IR + \frac{Q}{c}$

$\dot{Q} = I \longrightarrow \dot{V} = \dot{I} R + \frac{\dot{Q}}{c} = \dot{I} R + \frac{1}{c} I$

one more feature of L.T. : $\tilde{\dot{f}}(s) = s\, \tilde{f}(s) - f(0)$

$$\tilde{\ddot{f}}(s) = s^2 \tilde{f}(s) - s f(0) - \dot{f}(0)$$

take L.T. $[\dot{V} = \dot{I} R + \frac{I}{c}] = s\tilde{V} - V(0) = (s\tilde{I} - I(0))R + \frac{1}{c}\tilde{I}$

L.T. is linear $\widetilde{f+g} = \tilde{f} + \tilde{g}$, same for inverse

if $V(0) \equiv 0$ & $I(0) \equiv 0$, $s\tilde{V} = (sR + \frac{1}{c})\tilde{I}$

for sudden turn-on: $V(t) = V_0 \cdot U(t) \equiv$ V_0, $t=0$ $\longrightarrow t$

$V_0 s \tilde{U} = (sR + \frac{1}{C})\tilde{I}$

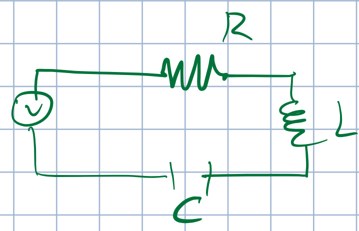
$$\tilde{I}(s) = \frac{1}{sR+\frac{1}{C}} \cdot s V_0 \tilde{\mathcal{U}} = \frac{V_0}{sR+\frac{1}{C}} = \frac{V_0/R}{s+\frac{1}{RC}} = \frac{V_0}{R}\cdot\frac{1}{s+\frac{1}{RC}}$$

$$= \frac{V_0}{R} \cdot \overbrace{e^{-t/RC}}$$

$$I(t) = \frac{V_0}{R} e^{-t/RC} \quad \text{for } t \geq 0$$

L.T. Ohm's Law

LCR circuit:



$$V = V_R + V_L + V_C$$

$$\tilde{V} = \tilde{V}_R + \tilde{V}_L + \tilde{V}_C$$

$$= \tilde{I}R + \tilde{\dot{I}}L + \frac{\tilde{Q}}{C}$$

$$= \tilde{I}R + L\cdot(s\tilde{I} - I(0)) + \frac{\tilde{Q}}{C} \qquad I(0) \to 0$$

$\tilde{\dot{Q}}$? $\qquad \dot{Q} = I \qquad \tilde{\dot{Q}} = s\tilde{Q} - Q(0) = \tilde{I} \qquad Q(0) \to 0$

$$\tilde{Q} = \frac{1}{s}\tilde{I}$$

$$\rightarrow \tilde{V}_C = \frac{1}{sC}\tilde{I}$$

$$\longrightarrow \tilde{V}_{drops} = \tilde{I}R,\ sL\tilde{I},\ \frac{1}{sC}\tilde{I}$$

$$\tilde{V} = \left[R + sL + \frac{1}{sC}\right]\tilde{I}$$

V is a switch

$$\tilde{I} = \frac{1}{R+sL+\frac{1}{sC}} \cdot (V_0 \tilde{\mathcal{U}})$$

$$\tilde{I} = \frac{V_0}{R+sL+\frac{1}{sC}} \cdot \boxed{\frac{1}{s}} = \frac{V_0/L}{s^2+\frac{R}{L}s+\frac{1}{LC}} = \frac{V_0/L}{\alpha}\left[\frac{1}{s+a} - \frac{1}{s+b}\right]$$

$a+b = \frac{R}{L} \qquad a\cdot b = \frac{1}{LC} \qquad \alpha = b-a$

$$a, b = \underbrace{-\frac{R}{2L}}_{\text{damping factor } \gamma} \pm i\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\longrightarrow \quad b - a = -2i \underbrace{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}_{\Delta \text{ driving freq}}$$

$$I(t) = \frac{V_0/L}{-2i\Delta}\left[e^{-at} - e^{-bt}\right]$$

$$= \frac{V_0/L}{-2i\Delta} e^{-\gamma t}\left(e^{-i\Delta t} - e^{i\Delta t}\right)$$

$$= \frac{V_0}{\Delta L} e^{-\gamma t} \sin(\Delta t)$$